Thermowell Design Methods and their Implementation in an Expert System

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Declaration

I hereby confirm that all the work in this report is my own and that all references have been clearly and correctly indicated.

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Summary

This research programme was carried out in order to establish design methods and procedures for thermowells and implement these methods in a computer-based expert system. Thermowells are accessories for the temperature measurement instrumentation and they have to protect the temperature sensor from hazardous environments at high pressures and temperatures. The sensor also needs protection from the flow of the fluid it is immersed in as it can cause the sensor to vibrate and subsequently damage it. Thermowells have to be designed in such a way that they can resist the vibration caused by the flowing fluid, do not collapse due to the surrounding pressure and the thermowell's material has to be compatible with the fluid to avoid corrosion of the thermowell. At the same time they have to transfer the heat of the fluid to the sensor to enable the temperature measurement.

A literature review was carried out to establish the available methods and procedures concerning thermowell design. The procedures that deal with the calculation of the natural frequency of thermowells, which is required to ensure the thermowell is not damaged by vibration, were verified using practical vibration analysis techniques.

After establishing the appropriate procedures to determine the vibration, pressure, stress and thermal aspects concerning thermowell design, they were implemented in an expert system. The software package KAPPA-PC was used to develop the application. After careful testing and elimination of different errors encountered during the testing phase, an expert system was delivered that carries out the analysis of a given thermowell design and also designs a thermowell for a specified application.

Contents

Nomenclature

Symbol	Definition	Units
Δ	deformation	m
ρ	density	kgm^{-3}
τ	time constant	s
τ_n	natural period of oscillation	s
υ	specific volume	m^3/kg
ω_n	circular frequency	rad/s
a	cross-sectional area	m^2
A	root diameter	m
	area	m^2
B	tip diameter	m
c	damping coefficient	N/ms
c_1, c_2, c_3, c_4	integration constants	-
C	constant	-
C_d	drag coefficient	-
C_K	Kármán-force coefficient	-
C_p	specific heat	J/(kgK)
d	bore diameter	m
d	differential	-
D	outside diameter of thermowell	m
E	modulus of elasticity	Nm^{-2}
F	force	N
F_A, F_B	stress correction factor	-
F_{W1}, F_{W2}	forces caused by thermowell weight	N
f_n	natural frequency	Hz
f_w	wake frequency	Hz
G	specific weight	kgm^{-3}
g	acceleration of gravity	ms^{-2}
J	second moment of area	m^4
k	spring stiffness	N/m
	thermal conductivity	W/(mK)
K_1, K_2, K_3	stress constants	-, m, -
K_a	constant	-
K_f	frequency constant	$in^{3/2}s^{-1}$
K_v	constant	-
K_x	constant	m^{-2}
L_1	length of thermowell completely immersed	m
L	immersion length	m
L_{max}	maximum allowable length	m
m	mass	kg
M	heat transfer factor	m^{-1}
$maxL$	maximum immersion length	m
$minL$	minimum immersion length	m
P	allowable pressure	bar
P_0	operating pressure	bar
q	uniformly distributed load	N/m

r	frequency ratio	-
s	reduced length	m
S	allowable stress	Nm^{-2}
Sr	Strouhal-number	-
t	time	s
	wall thickness at tip	m
T_a	ambient temperature	°C
T_b	temperature at sensing element	°C
T_e	temperature error	°C
T_f	temperature of process fluid	°C
U	overall heat coefficient	Wm^{-2}/K
v	velocity	m/s
V	flow rate	m^3/hr
W	gravitational force	N
W_1, W_2	weight of thermowell	kg
x	position along thermowell length	m
x_A, x_B	boundaries of area	m
x_G	centre of gravity	m
y	deflection of thermowell, beam, mass	m
y', y'', y''', y''''	first, second, third, fourth derivative of y	
y_{max}	maximum deflection	m
$\dot{y}$	velocity of mass	m/s
$\ddot{y}$	acceleration of mass	ms^{-2}

1. Introduction

Thermowells

Thermowells are used in power, process, pharmaceutical, chemical and petrochemical industries to protect temperature sensors and indicators which can be subject to damage by the application of high pressure or chemical attack. They also allow maintenance of the sensors to take place without interrupting the normal operation of the process or endangering the operator. Figure 1 shows a sketch of a typical thermowell installation.

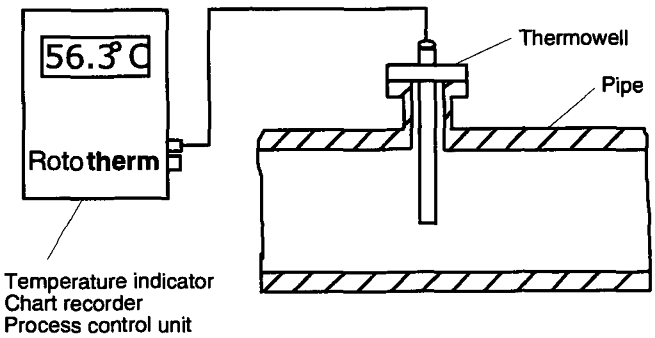


Figure 1: Typical thermowell installation

The thermowell is attached to the pipe with a matching flange. In most cases the fluid in the pipe, which can be a liquid or gas, is at an elevated temperature and under pressure; usually, the fluid is also flowing through the pipe. All these conditions have to be considered during the design process as they determine the fatigue life of the thermowell.

Failure of a thermowell can lead to considerable financial loss, harm to the environment and ultimately human life; therefore thermowells have to be designed specifically for a given application. An example for the importance of correct thermowell design is given in Appendix VI.

Different thermowells are shown in Figure 2.

Figure 2: Different thermowells

The characteristic dimensions of a thermowell as indicated in Figure 3 are:

- the root diameter, i.e. the outside diameter at the fixed end
- the tip diameter, i.e. the outside diameter at the free end
- the bore diameter
- the immersion length, i.e. the length from the fixed end to the free end

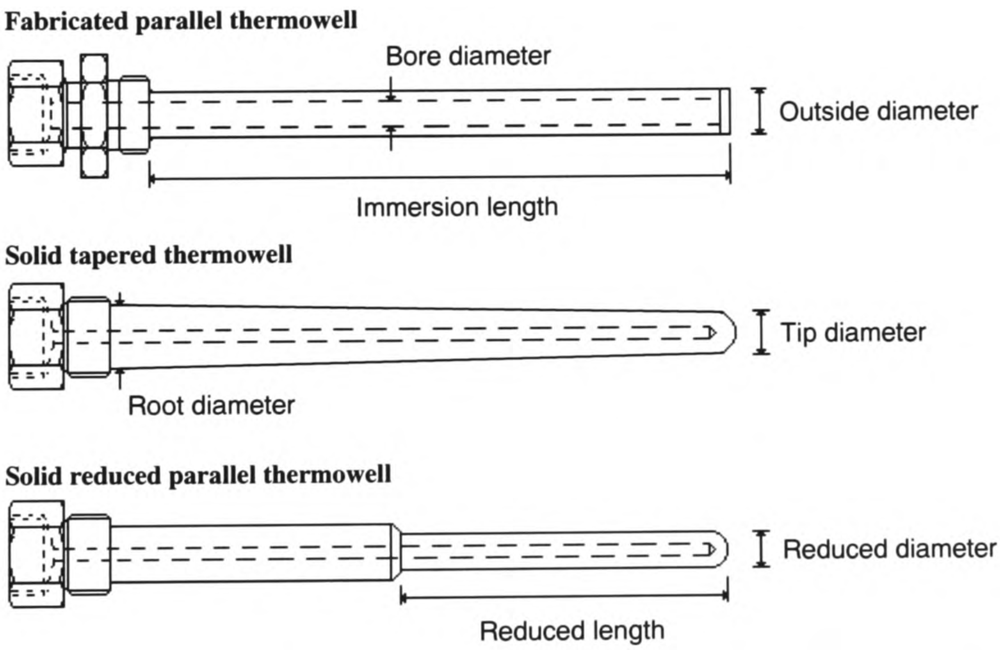


Figure 3: Basic thermowell shapes

Figure 4 shows a typical tapered thermowell.

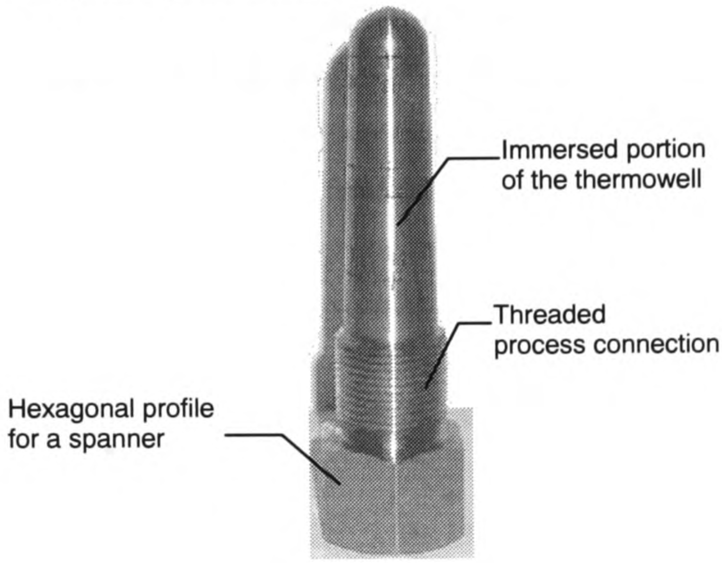


Figure 4: Typical tapered thermowell

Depending on the type of thermowell, some dimensions are constant, or others have to be introduced. In the case of a parallel thermowell for example, the root and tip diameter are equal and reference is usually made to the outside diameter. On the other hand, when specifying a reduced parallel thermowell, an additional length has to be introduced, indicating the length of the reduced part of the thermowell (see Figure 3).

There are two ways of manufacturing thermowells: drilled and turned from solid barstock, or fabricated from tubing. The latter is a cost effective and fast way of producing thermowells for low pressure applications; solid thermowells are more expensive and require more work in manufacturing, but they are stronger and can therefore be used at high pressures. Also, some more exotic materials are only available as barstock.

Expert Systems

Expert systems are computer programs that use a knowledge-based approach to solve problems in a narrow domain. They are used for applications where there is no definitive approach to establish a solution for the problem. The knowledge used by the expert system is that of human experts in a particular field and this knowledge is implemented in the software. The knowledge not only includes any necessary equations, but also 'rules of thumb' used by the human expert to achieve a solution for the given problem. These rules are usually IF...THEN... statements and the collection of all rules is called the *knowledge base*. The information in the knowledge base is used to solve the problems the expert system is designed for. The part of the expert system that is dealing with the operation of the expert system is called the *inference engine*. The inference engine is another major difference between expert systems and conventional programs as it establishes which rules are applicable for the current problem. This is contrary to conventional programs which would apply all the algorithms known to the system. This restricts the use of conventional programs to applications which allow the use of well defined procedures. If it is not possible to describe an exact solution path, it is necessary to search for solutions in a *solution space*; this requires the use of an expert system.

Collaboration between the University of Glamorgan and British Rototherm

The British Rototherm Co Ltd in South Wales is a manufacturer of industrial measurement equipment with a world-wide customer base. The product range includes mechanical and micro-processor controlled chart recorders, manometers, dial thermometers, temperature sensors (thermocouples, platinum resistance thermometers etc.), mechanical pressure gauges and pressure transmitters. New developments include digital thermometers and pressure gauges.

Rototherm had also identified an opportunity for increased sales in the thermometer and temperature sensor market by providing the customer not only with the sensor or thermometer itself, but also with the thermowell manufactured for the specific application.

Although British Rototherm had a production facility to manufacture a range of mechanical instruments and accessories, including thermowells, it lacked the necessary engineering expertise to carry out original design work. The designs for the mechanical instruments were inherited through acquisitions made by the Company. In order to develop a mechanical design capability at Rototherm a Teaching Company Scheme was set up between British Rototherm and the University of Glamorgan.

Thermowells may be manufactured according to customer designs, but even though the customer provides the design, a so-called 'wake frequency analysis' is required. This theoretical verification of the design was first introduced by J W

Murdock in 1959 and has been mandatory ever since; it has to be provided by the thermowell manufacturer. Thermowells should only be used in an application when they pass this analysis. Previously, the analysis had to be carried out by a subcontractor who would supply Rototherm with the results.

As part of the Teaching Company Scheme, design methods for thermowells had to be established and put in a useful form to provide a facility for routine in-house thermowell design and analysis.

Outline of work

In order to generate application specific designs of thermowells, appropriate procedures had to be established. Therefore, a review of the available literature on thermowell design and related topics was carried out, resulting in a collection of different equations and rules for the vibration, pressure, stress and thermal analysis of thermowells, together with material and process connection considerations. The methods concerning the vibration analysis, however, relate to tapered thermowells only. It was therefore necessary to establish an approach to determine the frequency relationships for parallel and reduced parallel thermowells. In order to verify the methods, practical vibration tests were carried out.

Following this, the possibility to implement the established design procedures in a computer-based expert system were investigated. Applying a number of criteria, a decision was made that expert system development would be suitable for the thermowell design application. Thereafter, the application development tool KAPPA-PC, which includes features necessary for expert systems, was chosen to aid the expert system development.

Subsequently, an expert system titled 'Thermowell Design Manager' was developed which carries out original thermowell design for a specific application and analyses existing thermowells. It also enables the estimation of the thermowell price. A database in the expert system contains relevant information about the materials, fluids and process connections which are necessary to carry out the designs.

Finally, the developed expert system was tested and debugged in order to deliver a stable and reliable application.

2. Design of Thermowells

2.1 Review of Published Work on Thermowell Design Procedures

In 1959 J W Murdock published the first paper dealing with a systematic approach for the design of thermowells. The paper was intended as the basis for a proposed ASME Power Test Code (PTC); the final Code (ASME 1974) is an almost exact copy of that paper, only a few changes have been made to the dimensions of the thermowell geometry.

Murdock identified three aspects which have to be considered when designing a thermowell: flow-induced vibration caused when a fluid is flowing over a thermowell, stresses in the thermowell caused by the vibration and external pressure, and variation of the material properties due to the process temperature.

Subsequent papers dealing with thermowell design also consider the effect of the thermowell on the measurement of the temperature in the process (Roughton 1965; Gibson 1995).

All of the papers concentrate on verifying that a thermowell is suitable for a given application. There are no suggestions of how to select a thermowell in the first place, or how to proceed if the thermowell fails the verification. However, in 1994 D Frank produced a paper concerned with the selection of thermowells. In his guidelines he suggests, as a starting point, to decide on the material used for the thermowell. The choice is restricted by the compatibility between the material and the process conditions, a consideration not clearly mentioned by other authors. He is also the only author to suggest criteria for the selection of the connection between thermowell and plant (usually referred to as process connection) according to certain conditions.

The methods in all the papers available suggest that a suitable design of thermowell is achieved by iteration; there is no definitive approach to thermowell design. In order to get an appropriate thermowell design, an initial design has to be proposed which takes given specifications into account. This will then be checked against the specifications using the available analysis methods discussed in the following chapters. If the design does not meet the requirements then changes to the design should be made and a new analysis carried out. This process has to be repeated until the proposed design meets all requirements.

The subsequent chapters deal with the vibration (section 2.1.1), stress and pressure (section 2.1.2), geometry (section 2.1.3), material (section 2.1.4), process connection (section 2.1.5) and thermal considerations (section 2.1.6) involved in the design of thermowells.

2.1.1 Vibration Analysis

To ascertain whether or not the effect of vibration will cause damage to the thermowell, the natural frequency of the thermowell has to be compared with the excitation frequency caused by the fluid flow (see also section 2.2.1.1.2 *Flow Induced Vibration*). This frequency is often referred to as the *wake frequency*. The relationship between the wake frequency f_w and the natural frequency f_n is called the *frequency ratio* r and is calculated using equation [1]:

$$r = \frac{f_w}{f_n} \qquad [1]$$

The frequency criterion is satisfied when the value of r is 0.8 or less. This means that the wake frequency should not exceed 80% of the thermowell's natural frequency, thus avoiding resonance. This relationship was first established by Murdock (1959), and the authors of other papers adopt this approach (Roughton 1965; Frank 1994; Gibson 1995). The limit of 0.8 for the frequency ratio was suggested to Murdock by Prof. J P Den Hartog during personal communications. There is no indication what this limit is based on. However, in the literature dealing with the effects of flow-induced vibration the so-called *lock-in effect* is mentioned (Harris 1994). When the flow velocity is increased or decreased so that the wake frequency approaches the natural frequency of the thermowell, the wake frequency can suddenly lock on to the natural frequency of the thermowell, therefore exciting the thermowell at its natural frequency and possibly causing resonance. It is assumed that Den Hartog suggested the 0.8 limit to avoid this effect. Experimental investigations carried out by Kassera *et al.* (1994) to confirm their computer simulations of flow induced vibrations of a cylinder in uniform cross flow revealed that "the excitation forces caused by the vortex separation oscillate with the maximum frequency of 18Hz, corresponding to 82% of f_n, so with lock-in-effect resonance could just be reached". This supports the 0.8 limit proposed by Den Hartog.

The generally accepted equation suggested by Murdock to calculate the wake frequency is:

$$f_w = 2.64 \cdot \frac{v}{B} \qquad [2]$$

Note that the equation in this form is valid only for a specific set of units, which is v (the flow velocity) in ft/s and B (the tip diameter of the thermowell) in inches. A discussion of how other units can be used is given in section 2.2 *Discussion of Design Methods*. The designs proposed by Murdock (1959) and in the PTC (ASME 1974) should only be used for velocities of up to 300ft/s, i.e. 91.44m/s. At higher velocities, considerable differences between measured and actual fluid temperatures have to be expected (Murdock 1959).

The natural frequency can be established using the equation

$$f_n = \frac{K_f}{L^2}\sqrt{\frac{E}{G}} \qquad [3]$$

where E and G are the modulus of elasticity and specific weight of the material, respectively, L is the length of the thermowell and K_f is the frequency constant which depends on the geometry (root, bore, tip diameter and length) of the selected thermowell; it can be found in a table in Murdock's paper (1959) and the PTC (ASME 1974). The table states values of K_f for five different sizes of thermowell (Size I - Size V) at various lengths. Murdock (1959) defines a size of thermowell as a tapered thermowell with a specific root, tip and bore diameter. In the final PTC, the table has changed slightly as geometries other than those suggested by Murdock (1959) have been used; the table is arranged according to the diameter of the sensing element used for temperature measurement. This dimension replaces the rather impractical thermowell 'size' used by Murdock (1959).

Equation [3] has been derived from a more complex equation to allow easier calculation of the frequency. However, using equation [3] requires the interpolation of K_f-values from the given PTC (ASME 1974)/Murdock (1959) tables if a thermowell with a geometry not listed is used. Roughton lists K_f-values for a few other geometries. It has to be noted that the proposed equation to establish the natural frequency can only be used for tapered thermowells. No author has described an approach to establish the natural frequency of the other common types, parallel and reduced-parallel thermowells.

To confirm his equation, Murdock (1959) carried out tests in the US Naval Boiler and Turbine Laboratory using thermowells of different sizes. The experiments showed a difference between calculated and measured natural frequency of 2.7% for Murdock's Size V thermowell to 15.3% for the Size I thermowells. The value for the difference is only given for thermowells of a length of six inches, because only for those thermowells an accurate measurement could be achieved at the time.

Vibration tests on flanged thermowells carried out by Blevins *et al.* in 1996 confirm Murdock's approach (1959). The main purpose of the tests, however, was to establish the effect damping has on the frequency response of thermowells. As a result of their work it is suggested that "heavy walled thermowells will not be damaged by resonant vibrations if the fluid has low density". A fluid is said to have low density if the density is $\rho \leq 2.7 kg/m^3$. Such a fluid would be steam at atmospheric pressure, for example, with a density of $0.263 kg/m^3$. However, at high pressures the density can be much higher: at 165 bar it is $43.57\ kg/m^3$. This shows that a decision about the likelihood of resonance cannot purely be based on the fluid used in the process; the pressure and temperature of the process have to be considered as well. It can therefore be argued that it is best to carry out the vibration analysis for all thermowell applications to avoid any complications during operation.

2.1.2 Pressure and Stress Analysis

The stress analysis carried out by Murdock (1959) is entirely based on procedures established by the US Naval Boiler and Turbine Laboratory. The appropriate report referenced by Murdock is unfortunately not obtainable. It is suggested that the stresses in the thermowell are caused by two pressures, the *static* pressure and the *velocity* or *dynamic* pressure. The location of the maximum stress is at the support point of the thermowell.

The static pressure is defined as the pressure of the fluid surrounding the thermowell; the velocity pressure is the *impingement force* divided by the projected thermowell area. The impingement force is caused by the fluid flowing past the thermowell and is calculated from the standard drag equation[1]

$$F_D = \frac{1}{2}\rho v^2 A C_D .$$

The drag coefficient C_D used in the equation is assumed to be unity.

The individual stresses created in the thermowell are (Murdock 1959):

- axial stress, caused by the thrust of the fluid pressure on the thermowell
- radial stress, produced by the fluid pressure
- shear stress, created as pure shear by the impingement of the fluid on the thermowell
- tangential stress, produced by the fluid pressure
- velocity stress, the bending stress caused by the impingement of the fluid

The analysis of the stresses showed that the shear stress can be neglected and that the maximum static pressure for safe use of the thermowell depends on the tip conditions (Murdock 1959). For practical use two equations were derived (Murdock 1959). Equation [4] enables the designer to calculate the maximum pressure a thermowell can withstand:

$$P = K_1 \cdot S \qquad [4]$$

where K_1 is a stress constant taken from a table in Murdock's paper (1959) or the PTC (ASME 1974), again for the five sensor and thermowell sizes, and S is the maximum allowable stress of the material used at the operating temperature. The constant K_1 can also be established for any thermowell geometry with the equation

$$K_1 = \frac{B^2 - d^2}{2 \cdot B^2 \cdot F_B} \qquad [5]$$

The stress correction factor F_B depends on the ratio ${}^{B-d}/_{2B}$ and has to be taken from a table in Murdock's paper (1959); there is no equation for its calculation. The value of F_B varies from 1.0 to 2.0. Murdock (1959) states that the thermowell design has to conform with the requirements of the ASME Code for Unfired Pressure Vessels. To achieve this conformity the factor F_B had to be introduced. It was established by comparing results achieved using an equation for the maximum static stress,

[1] Drag equation not given in Murdock (1959), equation from Bertin *et al.* (1998, p97)

$S = \frac{2 \cdot B^2}{B^2 - d^2} \cdot P_O$, with the stress-pressure graph in the ASME Code for Unfired Pressure Vessels (Figure UG-31 in the Code, according to Murdock (1959)).

Using equation [5], P can be calculated:

$$P = \frac{B^2 - d^2}{2 \cdot B^2 \cdot F_B} \cdot S \qquad [6]$$

Roughton (1965) proposes the use of the equation for the inner tangential stress, in re-arranged form:

$$P = \frac{B^2 - d^2}{2 \cdot B^2} \cdot S$$

The difference between the two equations for P is the factor F_B. As was stated when discussing Murdock's (1959) equation, the need for the factor arises from the conformity with the ASME Code for Unfired Pressure Vessels. As Roughton (1965) does not work to this Code, he uses the standard equation for the static stress, without the factor F_B.

In order to achieve a suitable design, the operating pressure has to be smaller than the calculated allowable pressure, i.e. $P_O < P$.

The second equation introduced by Murdock (1959) is used to calculate the maximum allowable length of the thermowell for given process conditions. The length of a thermowell is not only restricted by the frequency ratio between its natural frequency and the wake frequency, but also by stress considerations such as the bending stress caused by the fluid. Therefore, the maximum length can be calculated with the equation

$$L_{\max} = \frac{K_2}{v} \sqrt{\frac{\upsilon(S - K_3 P_O)}{1 + F_M}} \qquad [7]$$

K_2 and K_3 are constants given in the PTC (ASME 1974), v is the velocity of the fluid, S is the maximum allowable stress, P_O is the operating pressure, υ is the specific volume of the fluid and F_M is the *magnification factor* given by:

$$F_M = \frac{r^2}{1 - r^2}$$

with $r = \frac{f_w}{f_n}$.

Using this factor, the calculation of $L_{\max}$ will also consider the dynamic stress effects caused by the vibration of the thermowell.

Alternatively, the stress constants K_2 and K_3 can be established using the relationships:

$$K_2 = \sqrt{\frac{9262}{F_A \cdot K_x}} \qquad [8]$$

$$K_3 = F_A(K_a - 1) \qquad [9]$$

where: $K_x = \frac{1.698(A+2B)A}{A^4 - d^4}$

$K_a = \frac{A^2}{A^2 - d^2}$

F_A depends on $^{A-d}/_{2A}$ and has to be taken from the same table given in Murdock (1959) that is used for F_B.

Roughton (1965) does not use equation [7] to establish L_{max}. He combines the equation for the thermowell's natural frequency (equation [3]) with the frequency ratio r to give:

$$L_{max} = \sqrt{\frac{K_f}{1.25 \cdot f_w}} \cdot \left(\frac{E}{G}\right)^{\frac{1}{4}}$$

This equation does not take into account the fluid or the pressure the thermowell is exposed to or any stresses caused by the dynamic effects due to vortex shedding as identified by Murdock (1959). Roughton (1965) argues that for the application his paper is concentrating on (steam mains), the limit to avoid resonance is the more restrictive.

Using the equations for the natural frequency and the maximum length as proposed by Murdock (1959), this argument can be explained as follows:

In one case, the analysis will fail when checking the frequency ratio, i.e. r>0.8 whilst the length of the thermowell is shorter than the allowable length ($L<L_{max}$), therefore passing this part of the analysis.

Given other process conditions, especially a different fluid, the frequency ratio can be satisfactory (r<0.8), but the length of the thermowell is actually longer than acceptable, i.e. $L>L_{max}$. This is illustrated with an example:

Consider a thermowell being immersed into a pipe containing air, and one immersed into a pipe containing Propanol. Both fluids are at 25°C and atmospheric pressure and the flow has a velocity of 20m/s in both cases. The thermowells are made of the same material and have the same dimensions. Even though the frequency ratio between wake frequency and natural frequency is equal for both conditions (r=0.77), applying the stress analysis will show that the thermowell immersed in Propanol is too long for the application; the actual length of the thermowell of 210mm is larger than the allowable length of 189.5mm. The allowable length for the thermowell immersed in air is 4941mm (see Table 1). Increasing the immersion length to, say, 220mm would result in r=0.85, thus both wells would not pass the vibration analysis. However, the maximum allowable length for the thermowell immersed into air is L_{max}=2400mm, which is still larger than the actual immersion length. This shows that success or failure of a thermowell design not only depends on the geometry of the thermowell and the flow velocity, but also other process conditions such as the density of the fluid.

Table 1: Influence of fluid on thermowell design

Fluid	Air	Propanol
Density of fluid [kg/m³]	1.184	804.66
Natural frequency [Hz]	316.6	316.6
Wake frequency [Hz]	244.4	244.4
Frequency ratio [-]	0.77	0.77
Vibration criterion satisfied	Yes	Yes
Allowable pressure [bar]	580.1	580.1
Pressure criterion satisfied	Yes	Yes
Allowable length [mm]	4941	189.5
Length criterion satisfied	Yes	No

It is assumed that Roughton (1965) based his equation for L_{max} on the assumption that fluids with an appropriately low density are used in the process. Therefore Roughton (1965) suggested his approach to avoid designing thermowells that will pass the stress analysis but fail the vibration test. To compare this approach with Murdock's suggestion (1959), a thermowell was analysed using both methods. As K_f is required for the calculations, a thermowell geometry suggested by Murdock (1959) was used to avoid interpolation. The process conditions are the same as in the previous example (see *Appendix I* for calculation of the tabulated values).

Thermowell size IV, A=1", B=$^{13}/_{16}$", d=$^{7}/_{16}$", L=10" ➔ K_f = 2.661in$^{3/2}$/s

The natural frequency f_n, wake frequency f_w and therefore the frequency ratio r are equal for both fluids (Air and Propanol). However, the maximum allowable length is different for the fluids and the two calculation methods.

Table 2: Comparison between Murdock and Roughton approach for Lmax

Fluid	Air	Propanol
Density of fluid [kg/m^3]	1.184	804.66
Natural frequency [Hz]	267.78	267.78
Wake frequency [Hz]	213.204	213.204
Frequency ratio [-]	0.797	0.797
Vibration criterion satisfied	Yes	Yes
Allowable length (Murdock) [mm]	5356	205
Length criterion satisfied	Yes	No
Allowable length (Roughton) [mm]	255	255
Length criterion satisfied	Yes	Yes

Looking at the results given in Table 2, it can be seen that the length criterion is satisfied for both approaches when considering the thermowell in air. However, when analysing the thermowell in Propanol, the analysis will show that the thermowell fails when using Murdock's approach, which suggests that fluids with a high density are more critical in thermowell applications than fluids with a lower density. This confirms the observations by Blevins *et al.* (1996).

Gibson (1995) clarifies that the equation for the maximum length of the thermowell proposed by Murdock (1959) is based on an infinite fatigue life and that the velocity used in the equation is the maximum velocity, i.e. the centreline velocity of the flow and not the average velocity.

Finally, Roughton (1965) also investigated the effect of thermal stresses. These stresses are caused by a temperature variation between the thermowell tip and the root or the pipe wall. The variation can be quite significant, for example when the fluid is first introduced into the pipe or tank. The temperature differences cause different expansion rates along the length of the thermowell, which in turn can give rise to significant stresses. Because thermal stresses are transient and are only of importance when pressure stresses can be neglected, these stresses can be omitted when designing a thermowell.

2.1.3 Geometry and Manufacture

The geometry and manufacturing process is also an important aspect in thermowell design. The simplest shape would be a cylindrical thermowell as it is easiest to machine. This design can be even more simplified by using readily available tubes, which will be parted off to the appropriate immersion length. Then, a pre-machined adapter which provides both the process and instrument connection, and a disc which will close off the free end are welded to the tube, resulting in a fabricated thermowell. This is the simplest and most cost effective way of thermowell manufacture. However, the use of such a thermowell is restricted to pressures of less than 70 bar (Rototherm 1996). For higher pressures, thermowells machined from solid bar stock are used. Because both the outside and bore diameters have to be machined, it is possible to adjust the wall thickness more freely than for fabricated thermowells to suit the given process pressure. On the negative side this type of thermowell is more costly, because of the increased machining operations and the higher material costs.

Murdock suggested a tapered thermowell shape to reduce the stresses at the support end of the thermowell, while still keeping the shape rather simple (Murdock 1959). He states that an ideal shape would be such that the stresses are equal at all cross-sections; such a shape is impractical to fabricated, however. In other papers, the possible thermowell geometries are not discussed and the tapered shape proposed by Murdock is used in the discussions (Gibson 1995; Roughton 1965). D Frank (1994), however, suggests to choose the thermowell profile, together with the bore diameter, according to the temperature sensor used. Therefore, tapered thermowells should be used with thermocouples because they are tip sensitive and the best heat transfer occurs at the tip of the tapered thermowells due to the small wall thickness. Reduced parallel thermowells are recommended for RTDs (resistance temperature detectors) because these temperature detectors are stem sensitive. For these detectors it is also recommended to choose a bore diameter which is only slightly larger (Frank (1994) suggests 0.01"=0.25mm) than the stem diameter to improve the heat transfer and therefore measurement accuracy and response time. However, if a tapered profile is not required due to the vibration, pressure and stress considerations, a parallel thermowell shape can be used for either instrument because the thermowell's wall thickness is at its minimum along the whole immersion length.

In Jones (1985, p62) it is also suggested to use an immersion length which is at least three times larger than the length of the sensing area of the temperature sensor. This will aid in reducing the measurement error caused by conduction.

2.1.4 Material Considerations

The selection of the material is an important point when designing a thermowell. As Frank (1994) points out, it should be the first step in the design process. The thermowell material has to be selected according to the process conditions, i.e. the process temperature, pressure and the fluid the thermowell is immersed into. The material has to be chemically resistant to the fluid to avoid corrosion, pitting or any other chemical reactions. Also, the maximum possible operating temperature must be at least as high as the process temperature to avoid poor mechanical properties of the material. These properties are critical for the vibration, pressure and stress considerations and a poor material selection will cause one, or more, of these criteria to be unsatisfied.

The thermal characteristics of the material must also be considered. Thermowells are used in temperature measurement, and their other objective next to protecting the temperature sensor is to transport heat from the process to the sensor. Therefore, a thermally conductive material must be chosen; insulating materials cannot be used for thermowell design.

The most common materials used for thermowells are 316 and 304 grades of stainless steel; they exhibit good corrosion resistance and therefore offer a wide application range. In high temperature applications, nickel-chromium alloys such as Inconel (75% nickel, 16% chromium, 8% iron) can be used. Nickel-molybdenum alloys such as Hastelloy[2] (16% chromium, 16% molybdenum, 5% iron, 4% tungsten, balance nickel) are used when a high resistance at high temperatures is required, while nickel-copper alloys such as Monel (31% copper, balance nickel) are used in marine applications due to their good resistance to saltwater corrosion.

[2] Trade name of the Cabot Corporation

Frank (1994) lists a small selection of materials, specifying their application and maximum service temperature but none of their mechanical or thermal properties; Richmond (1980) specifies thermal characteristics of some materials but does not indicate their application areas. The ASME PTC 19.3 (ASME 1974) just states that any material approved by the ASME Boiler and Pressure Vessel or Piping Codes can be used.

There is also an increased demand for stainless steel thermowells with corrosion resistant protective coatings such as Teflon (OMEGA1981), Tantalum, PTFE or Gore Fluoroshield (Rototherm 1996) which are used in harsh chemical environments.

Information on the properties of a material can be obtained from textbooks, which usually specify an average value, or from the material suppliers. Books on heat transfer specify the necessary values required for thermal considerations for both the thermowell material and the process fluid, whilst information on material compatibility and corrosion can be found in material databases such as Fulmer (1979).

2.1.5 Selection of Process Connection

A suitable connection has to be chosen to attach the thermowell to the process pipe or vessel. The ASME recommendation on this issue is that "any manner approved by the Boiler and Pressure Vessel and Piping Codes" can be used (ASME 1974). It is important for the connection to prevent leaks. Generally, there are three types of suitable process connections: threaded, flanged or welded.

Threaded connections are used in small pipes where the well will not be removed frequently and corrosion is not a serious problem (Frank 1994). The thermowell is threaded into a boss directly welded to the pipe or vessel.

Flanges are more commonly used for high pressure applications (depending on the flange rating) and in larger pipes. They are also used when the thermowell has to be removed on a regular basis due to corrosion or other requirements (Frank 1994). The flanged connection is manufactured from a blind flange which is machined to provide a bore through which the thermowell passes. The thermowell is attached to the flange with either a fillet weld or a full penetration weld (for increased safety, but with considerable higher cost).

In high temperature, high pressure installations where removal of the thermowell is not required, welded connections are used. The thermowell is directly welded into the pipe/vessel, similar to the fitting required for threaded connections (Frank 1994).

The appropriate process connection can be selected according to these guidelines.

2.1.6 Thermal Considerations

Thermowells are used in order to protect the temperature sensor from the conditions of the process the sensor is used in. At the same time, however, the thermowell has to transfer heat from the process to the sensor, otherwise it is not possible to measure the temperature of the process fluid - the sole reason why a temperature sensor is required in the first place. The thermowell must have a certain wall thickness in order to protect the sensor, however. It is therefore necessary to estimate the effect the thermowell has on the temperature measurement.

The thermowell can introduce uncertainties to the temperature measurement if the measured temperature is not equal to the process temperature (Benedict and Murdock, 1963) and can therefore degrade the measurement accuracy (Jones 1985). This measurement error is a combination of several individual errors (Roughton 1965; Benedict and Murdock 1963; Murdock and Fiock 1950). First, the required fluid temperature has to be defined. When a fluid is flowing through a pipe then the fluid's temperature varies with the radial distance from the pipe walls, similar to the variation of the flow velocity (Roughton 1965). The required temperature for the measurement is defined as the mean temperature of fluid passing through a given cross-section of pipe (Roughton 1965). To achieve this requirement, the tip of a thermowell has to be at a position where the measured temperature equals the mean temperature. Such a definition and requirement has not been found elsewhere, and it can be assumed that it is used for the application discussed in Roughton's paper (1965).

The different errors which are combined to give the overall measurement error have been identified by Roughton (1965) as the *gradient error*, the *profile error*, the *pocket conduction error*, the *radiation error*, the *intrinsic dynamic error*, the *velocity equivalent error* and the *couple conduction error*. Murdock and Fiock (1963) only identify three errors, i.e. the *convection error*, the *radiation error* and the *conduction error*, as they only carry out a steady-state analysis which does not involve temperature changes. Richmond (1980) considers only the conduction and convection effects under the assumption that radiation can be neglected for any fluid and for gases and vapours at temperatures lower than 120°C. At higher temperatures, radiation can be significant. He also states a calculation method for the time constant of the thermowell. Gibson (1995) uses the same argument.

Richmond (1980) assumes that heat energy flows across the surface between the thermowell and the process fluid by convection only. This assumption is true in any liquid. In gases and vapours at temperatures higher than 120°C, however, heat transfer by radiation may be significant, too.

Richmond's equation for the measurement error is based on the heat transfer into and out of the thermowell. If other temperature effects can be neglected, then the heat transfer must be in balance.

The heat transfer by convection can be written as

$$Q_2 = UA(T_f - T_w)$$

where U is the overall heat coefficient, A is the thermowell surface area and T_f and T_w are the temperature of the fluid and the thermowell, respectively.

Heat also flows longitudinally along the cross section of the thermowell. This form of heat transfer can be assumed to be conductive:

$$Q_1 = \frac{ka(T_w - T_a)}{144L_1}$$

where k is the thermal conductivity of the thermowell material, a is the cross-sectional area, T_a is the ambient temperature and L_1 is the length of the thermowell immersed in the flowing region of the fluid. Gibson (1995), who uses the same equations, does not make a distinction between L and L_1.

Applying the heat balance $Q_1(x) = Q_1(x+\mathrm{d}x) + Q_2$ as illustrated in Figure 5

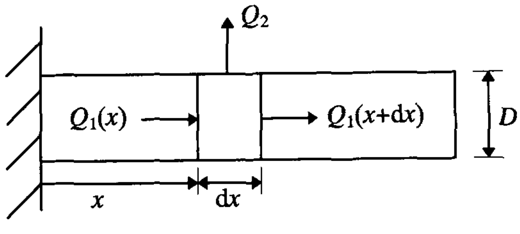


Figure 5: Differential thermowell element for the heat balance (Richmond 1980)

results in equation [10]:

$$T_e = \frac{T_f - T_a}{\cosh(ML)} \quad [10]$$

$$\text{with } M = \sqrt{\frac{\pi DU}{12ka}}$$

for the calculation of the measurement error. M is referred to as the heat transfer factor.

The equation for the time constant is given as

$$\tau = \frac{mC_p}{UA} \quad [11]$$

where C_p is the specific heat of the thermowell material and m is the mass of the thermowell.

Neither Richmond (1980) nor Gibson (1995) indicate a method of establishing the overall heat coefficient. However, Roughton states that the worst case overall heat coefficient is for a static fluid, v = 0 m/s. He also lists a few examples for the coefficient.

The temperature errors identified by Roughton (1965) are defined as follows: the gradient error δT_g is the difference between the mean temperature and the local temperature $T(r)$ at the tip of the thermowell due to the temperature profile. Figure 6 shows a typical temperature profile in a pipe, together with the mean temperature T_m.

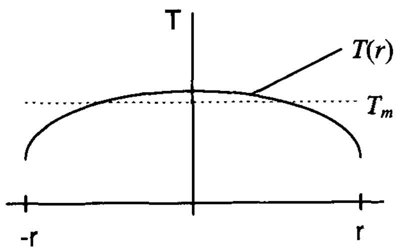


Figure 6: Temperature profile T(r) and mean temperature Tm

If all other errors are neglected at this stage, then the temperature indicated depends only on the length of the thermowell. But as stated by Roughton (1965), the required temperature is the mean temperature, which can only be measured with a thermowell with one of two possible immersion lengths L_1 and L_2, see Figure 7.

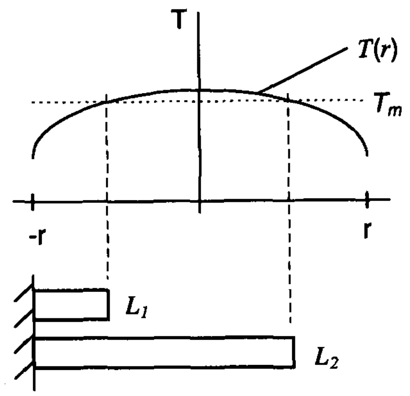


Figure 7: Possible immersion lengths to measure the mean temperature

Choosing any other immersion length will indicate a temperature higher or lower than the mean temperature, thus causing the gradient error. This requirement of measuring the mean temperature has only been stated by Roughton (1965). To determine the gradient error, a graph is provided by Roughton (1965). The value taken from the graph has to be multiplied by a factor θ which is calculated from

$$\theta = \frac{2.86 f \rho v_m C_p}{h_{sp}}$$

where v_m is the mean velocity, h_{sp} is the heat transfer coefficient for forced convection between the fluid and the pipe wall and f is the friction factor for the pipe. The friction factor has to be taken from reference books, none of which have been mentioned in the paper, and has to be defined according to the definition by Roughton (1965), $f=2F/\rho v_m^2$. F is the frictional force per unit area of wetted pipe surface.

The profile error represents the difference in local temperature at the tip of the thermowell due to departure of the temperature profile from the normal distribution caused by varying pipe geometry, for instance, bends or valves. This error is caused by the installation and therefore cannot be considered during the design of a thermowell.

The pocket conduction error δT_c is the difference between the tip temperature and the local steam temperature due to loss or gain of heat by conduction along the thermowell to or from the pipe walls. The equation for the calculation of the error,

$$\delta T_c = -\frac{\Delta T_s}{\cosh ML},$$

is similar to equation [10] proposed by Richmond (1980) to establish the measurement error. In the equation for δT_c, L is the immersion length of the thermowell and ΔT_s is the temperature difference between the pipe wall and the local temperature. Note that Roughton (1965) uses the ambient temperature and not the pipe wall temperature, which can be considerably different if the pipe is insulated. M is referred to by Roughton (1965) as the pocket conduction error parameter and is calculated with

$$M = \sqrt{\frac{\pi D h_s}{ka}}.$$

This is the same equation as given by Richmond (1980), except that the local heat transfer coefficient for forced convection between fluid and thermowell h_s is used instead of the overall heat coefficient U.

The radiation error δT_r is caused by the heat exchange through radiation between the tip of the thermowell and the pipe walls. To calculate this error, the equation

$$\delta T_r = \frac{h_r(1-\alpha_s)\Delta T_s + (\alpha_{sp} - \alpha_s)\varepsilon_p \sigma T_p^4}{h_s + h_r}$$

is used, where α_s is the absorptivity of the fluid at fluid temperature, α_{sp} is the absorptivity of the fluid at pipe wall temperature, ε_p is the emissivity of the thermowell and σ is Stefan's constant. h_r, the heat transfer coefficient for radiant heat exchange between thermowell tip and pipe wall, is calculated with

$$h_r \cong \frac{\varepsilon_p \sigma (T_s^4 - T_p^4)}{\Delta T_s}.$$

Presumably, Stefan's constant σ refers to the Stefan-Boltzmann constant with a value of $\sigma = 5.670 \cdot 10^{-8}$ $Wm^{-2}K^{-4}$, which is used in radiation problems (Incropera and DeWitt 1996, p9). According to Roughton (1965), the emissivity of thermowells is not easily estimated and can change due to surface oxidation and corrosion. However, he states that the value can be assumed to be 0.8-0.9. A similar problem is encountered with the radiation characteristics, especially for steam at elevated temperatures. If it is not possible to establish values for the characteristics of steam, then the simplification $\alpha_s = \alpha_{sp} = 0.6$ can be used (Roughton 1965).

The intrinsic dynamic error δT_d is the difference between the sensor temperature and the local fluid temperature arising from thermal inertia of the thermowell/sensing element combination when the fluid temperature is changing. It is determined by multiplying the intrinsic dynamic error coefficient Y with the rate of change of the fluid temperature, i.e. $\delta T_d = -Y\dot{T}_{sm}$. The intrinsic dynamic error coefficient is calculated by adding up the time constants for the thermowell and the temperature sensor. The time constant for the thermowell is calculated in the same manner as proposed by Richmond (1980). According to Roughton (1965), the time constant for a thermocouple can be calculated using the equation $\tau_t = {}^{C}/_{\pi dh}$, where C is the thermal capacity and h is the combined coefficient for heat transfer by conduction and radiation from the thermowell wall to the thermocouple sheath, across the air gap and then through the insulation to the thermocouple itself.

When measuring temperature in a moving fluid, additional kinetic energy in the direction of the flow is created which is equivalent to an increase in temperature called the *dynamic temperature*. The total temperature in a flowing fluid is therefore the sum of the static temperature and the dynamic temperature. However, the fluid impinging on the thermowell is not completely brought to rest, which results in only a part of the dynamic temperature being measured. The proportion measured is referred to as the recovery factor of the thermowell. Roughton (1965) calls the part of the dynamic temperature not being measured the velocity error δT_v and provides the equation

$$\delta T_v = \frac{(1-Q)v^2}{2gJC_p}$$

for its calculation. In the equation, Q is the recovery factor and $J = 778.16 lb_f ft/Btu$ is the mechanical equivalent of heat[3]. According to Roughton (1965), the recovery factor has to be established experimentally and the velocity error is therefore difficult to consider when designing a thermowell.

The couple conduction error δT_t is the difference between the temperature at the tip of the thermowell and at the thermocouple junction due to loss or gain of heat by conduction along the thermocouple, similarly to the conduction along the thermowell itself (see *pocket conduction error*).

[3] The value for J is taken from Incropera and DeWitt 1996, p26

An equation proposed to estimate that error is

$$\delta T_t = T_1 - T_t = 2m_2 \frac{(T_1 - T_2)}{m_1 + m_2} e^{-m_1 L}$$

L is the length of the thermocouple that is surrounded by a constant temperature T_1, the remainder of the thermocouple is in a region of temperature T_2. In the case of thermowells, T_1 is taken as the tip temperature of the thermowell and T_2 is the ambient temperature, i.e. the temperature outside the pipe. m_1 and m_2 are the thermocouple conduction error parameters which are calculated from

$$m = \sqrt{\frac{h}{k_t w}}$$

with w being the wall thickness of the thermocouple sheath and h and k_t are the thermocouple's heat transfer coefficient and thermal conductivity, respectively. According to Roughton (1965), this error is generally small and can be eliminated by providing a close fit between the thermocouple and the thermowell's bore.

The overall error introduced by the thermowell and the thermocouple is estimated by adding up the individual errors. It is therefore

$$\delta T_n = \delta T_g + \delta T_c + \delta T_r + \delta T_v + \delta T_d + \delta T_t$$

The reasons for a difference in measured and actual fluid temperature identified by Benedict and Murdock (1963) are that the pipe wall temperature is different from the fluid temperature, the temperature of the root of the thermowell is different from the temperature at the thermowell's tip, the fluid is at a velocity relative to the thermowell and the sensor could be improperly calibrated. The latter reason cannot be considered in thermowell design, however.

It is argued that the thermowell receives heat by convection and radiation and loses heat by conduction and radiation. This approach is similar to the proposal by Richmond (1980) but it also includes the radiation effects. The equation for the heat balance of a differential thermowell element in this case is

$$\mathrm{d}q_c = \mathrm{d}q_r + \frac{\mathrm{d}q_k}{\mathrm{d}x}\mathrm{d}x$$

and the differential element is illustrated in Figure 8.

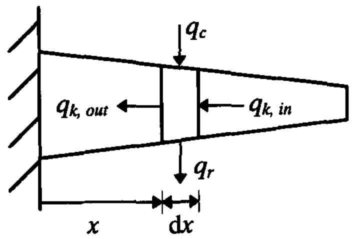


Figure 8: Differential thermowell element for heat balance (Benedict and Murdock 1963)

In both the equation and Figure 8 q_c is the convective rate of heat transfer, q_k is the conductive rate of heat transfer and q_r is the heat transfer through radiation.

The heat transfer rate through convection can be calculated using the equation

$$\mathrm{d}q_c = h_c \mathrm{d}A_c (T_{adi} - T_x)$$

T_{adi} is referred to as the adiabatic temperature and is calculated from

$$T_{adi} = T_s + \frac{\alpha v^2}{2JgC_p}$$

The adiabatic temperature is the total temperature mentioned by Roughton (1980) which is the sum of the static temperature T_s and the dynamic temperature caused by the impingement of the fluid on the thermowell (see previous pages on Roughton's approach for more detail). α is the recovery factor (Q in Roughton's equation), A_c is the surface area of the thermowell, T_x is the thermowell temperature of the differential element under consideration and h_c is the convective heat transfer coefficient.

The convective heat transfer coefficient h_c can be calculated using the equation for the Nusselt number:

$$\mathrm{Nu} = a\,\mathrm{Re}^b\,\mathrm{Pr}^c = \frac{h_c D}{k}$$

This is similar to the approach suggested by Gibson (1995), who proposes the equation

$$\mathrm{Nu} = C\,\mathrm{Re}^n\,\mathrm{Pr}^{1/3}$$

Benedict and Murdock (1963) state that values for the constants a, b and c can be found in the literature, but they also list possible values from different sources. Gibson also gives values for the constants C and n for different values of the Reynolds number Re. Gibson's approach can also be found in Incropera and DeWitt (1996), p344, together with appropriate values for the constants. Looking at the values for constant c given in Benedict and Murdock, it can be seen that the values are either 0.3 or 0.31, which is close to $^1/_3$ as suggested in Gibson's equation. Also, the values for a and b are comparable with the values for C and n, respectively. Therefore, either of the two approaches can be used to establish h_c.

The Reynolds number Re is calculated with $\mathrm{Re} = {}^{dv\rho}/_{\mu} = {}^{dv}/_{\nu}$ (Bohl 1991, p114), the Prandtl number is $\mathrm{Pr} = C_p{}^{\mu}/_k$ (Incropera and DeWitt 1996, p310).

According to Benedict and Murdock (1963) there are several recovery factors that can be used for α. It is claimed that the adiabatic recovery factor, which remains constant around the periphery of the thermowell, is identical with the flat-plate recovery factor $\alpha_{\text{laminar}} = \mathrm{Pr}^{1/2}$ and $\alpha_{\text{turbulent}} = \mathrm{Pr}^{1/3}$ and is therefore convenient to use.

The heat transfer rate through radiation is calculated using the equation

$$\mathrm{d}q_r = h_r \mathrm{d}A_r (T_x - T_w)$$

$$\text{with } h_r = \frac{\sigma\varepsilon'(T_x^4 - T_w^4)}{T_x - T_w}$$

$$\text{and } \varepsilon' = \varepsilon_x \left[1 + \frac{\varepsilon_{f,x}(F-1)T_x^4 - \varepsilon_{f,f} F T_{adi}^4 + \varepsilon_{f,w} T_w^4}{T_x^4 - T_w^4} \right]$$

T_w is the temperature of the pipe wall, F is the adjusted emissivity calculated from $F = (\varepsilon_x + 1)/2\varepsilon_x$, A_r is the surface area of the thermowell, ε is the emissivity of the thermowell and $\varepsilon_{f,x}$ is the emissivity of the fluid at thermowell temperature, $\varepsilon_{f,f}$ of the fluid at fluid temperature and $\varepsilon_{f,w}$ of the fluid at wall temperature. Benedict and Murdock (1963) indicate that there is a lack of data concerning the emissivity of gases and suggest to use the simplification $\varepsilon_{f,x} = \varepsilon_{f,f} = \varepsilon_{f,w}$.

Finally, heat is transferred from the tip of thermowell to its root by means of conduction. The heat rate through conduction is calculated from Fourier's conduction equation

$$q_k = -kA_k \frac{\mathrm{d}T_x}{\mathrm{d}x}$$

to be

$$\frac{\mathrm{d}q_k}{\mathrm{d}x}\mathrm{d}x = -kA_k \frac{\mathrm{d}^2 T_x}{\mathrm{d}x^2}\mathrm{d}x - k\frac{\mathrm{d}T_x}{\mathrm{d}x}\frac{\mathrm{d}A_k}{\mathrm{d}x}\mathrm{d}x$$

as required in the heat balance equation, where A_k is the cross-sectional area of the thermowell.

Using the established equations for q_c, q_r and q_k the heat balance equation can be written as

$$\frac{d^2T_x}{dx^2} + a_1(x)\frac{dT_x}{dx} - a_2(x,y)T_x = -a_2a_3(x,y)$$

where $$a_1(x) = \frac{dA_k}{A_k dx}$$

$$a_2(x,y) = \frac{dA_c(h_r + h_c)}{kA_k dx}$$

$$a_3(x,y) = \frac{h_c T_{adi} + h_r T_w}{h_c + h_r}$$

This is a second order, first-degree, non-linear equation which has no known closed form solution. Benedict and Murdock (1963) suggest three possible approaches to solve the equation: the overall linearisation, the tip solution where all conduction effects are neglected, and the stepwise linearisation. By their own account the stepwise linearisation produces the most accurate results. Using this method, the thermowell is divided lengthwise in a number of elements as illustrated in Figure 9.

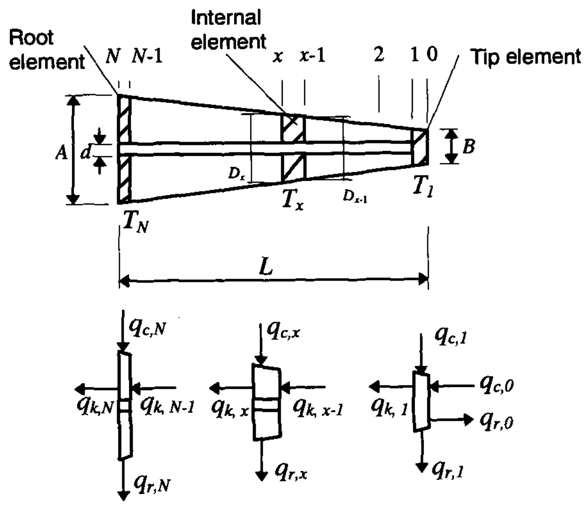


Figure 9: Thermowell elements for stepwise linearisation

The temperature T_x represents the temperature of that entire element. This means that the accuracy of the result increases with an increasing number of elements. Three individual heat balance functions are necessary to describe the complete thermowell; one equation is required to represent the tip, one for the root and one equation for the elements between the root and tip of the thermowell. It is possible to solve these equations for the temperature in each element of the thermowell. This gives the following equations:

- tip element

$$T_2 = T_1 - \frac{A_{c,0} + A_{c,1}}{A_{k,1}} \frac{\Delta x}{k} \left[h_c T_{adi} - (h_c + h_{r,1}) T_1 + h_{r,1} T_w \right]$$

- internal element

$$T_{x+1} = \left(1 + \frac{A_{k,x-1}}{A_{k,x}}\right) T_x - \frac{A_{k,x-1}}{A_{k,x}} T_{x-1} - \frac{A_{c,x}}{A_{k,x}} \frac{\Delta x}{k} \left[h_c T_{adi} - (h_c + h_{r,x}) T_s + h_{r,x} T_w \right]$$

- root element

$$T_{w'} = \left(1 + \frac{A_{k,N-1}}{2A_{k,N}}\right) T_N - \frac{A_{k,N-1}}{2A_{k,N}} T_{N-1} - \frac{A_{c,N}}{2A_{k,N}} \frac{\Delta x}{k} \left[h_c T_{adi} - (h_c + h_{r,N}) T_N + h_{r,N} T_N \right]$$

In order to solve the thermal problem, an initial temperature has to be assumed for T_1. The other temperature T_2, T_3, ..., T_N and $T_{w'}$ are then calculated using the above equations. The calculated pipe wall temperature $T_{w'}$ is then compared with the given wall temperature T_w and adjustments are made to T_1 for another try in case T_w and $T_{w'}$ are different. This iterative approach will soon give a solution. The measurement error is the difference between the adiabatic, or total, temperature T_{adi} and the temperature T_1 that results in $T_{w'} = T_w$, i.e. $T_e = T_{adi} - T_1$.

2.2 Discussion of Design Methods

Having established all the relevant issues concerning thermowell design, a decision has to be made as to which methods have to be implemented and in what way. It is also necessary to investigate if the established equations can be used without any modifications when using a different set of units than suggested in the respective source. Most of the equations found use imperial (inch, lb etc.) units. Nowadays, engineering units are mostly SI-units - a practice also used at British Rototherm. Therefore, the equations' compatibility with different sets of units has to be established and, if required, the equations have to be rewritten so that they can be used with SI-units. This check is required because the values for some equations have to be specified in inconsistent units. For example, the equation for the wake frequency (Murdock 1959) requires the flow velocity in ft/s and the diameter in inches. Nowadays it is common practice to formulate equations is such a way that any system of units can be used as long as the units remain consistent throughout. For instance, if the flow velocity is given in m/s then the diameter has to be specified in m; if the flow velocity is given in miles/hour then the diameter has to be in miles and so forth. This seems not to be the case for the equations originally specified by Murdock, therefore this 'unit compatibility check' is essential.

2.2.1 Vibration Analysis

All the relevant papers on thermowell design use the method of analysing a thermowell's vibration criterion proposed by Murdock (1959). This involves establishing a ratio between the wake or vortex shedding frequency and the thermowells natural frequency. If the ratio is smaller than 0.8 then the thermowell has passed the wake frequency analysis. Even though Blevins *et al.* (1996) have argued that when a thermowell is immersed into a flow of low density the thermowell will not be damaged even at resonance, i.e. r=1, there have been no suggestions for a different approach.

Carrying out the vibration analysis according to Murdock's method is common practice in the USA; it is part of the ASME PTC19.3 (ASME 1974). But this method is also used throughout the process industry in Britain, with customers specifically asking for the design or analysis to be carried out according to ASME PTC19.3. With this commercial aspect in mind and the knowledge that the method derived by Murdock is based on good engineering work, it was decided to adopt this method for implementation in the expert system.

However, this approach is only valid for tapered thermowells. There are no suggestions in either the PTC (ASME 1974) or any other thermowell related papers on how to establish the natural frequency for parallel or reduced parallel thermowells, which is essential when carrying out the vibration analysis.

It was therefore decided to review Murdock's (1959) original derivation of equation [3] and to develop mathematical models of parallel and reduced parallel thermowells. These models would then be validated using practical vibration analysis methods.

Before establishing equations for the natural frequency of different thermowells, the equation for the wake frequency f_w (equation [2]) has to be checked for its validity when using different units. As was indicated earlier, equation [2] can only be used in the given form when using units of ft/s for the flow velocity and inches for the outside diameter. The requirement for these units suggests that a conversion from inches to feet (or vice versa) is carried out in the specified equation. The relationship between feet and inches is 1 ft = 12 in. Together with the standard equation for the wake frequency (see section 2.2.1.1.2 *Flow Induced Vibration*), the units suggested by Murdock (1959) and a Strouhal-number of Sr = 0.22, the relationship for the wake frequency can be written as (units in [ ]):

$$f_w = 0.22\frac{v}{B} \qquad \left[\frac{\mathrm{ft}/\mathrm{s}}{\mathrm{in}} = \frac{\mathrm{ft}/\mathrm{s}}{\mathrm{ft}/12}\right]$$

$$f_w = 0.22 \cdot 12 \cdot \frac{v}{B} \qquad \left[\frac{\mathrm{ft}/\mathrm{s}}{\mathrm{ft}} = 1/\mathrm{s}\right]$$

$$f_w = 2.64\frac{v}{B} \qquad \left[1/\mathrm{s} = \mathrm{Hz}\right]$$

This clearly shows that the necessary conversion of the units is taken care of in the equation when using the units suggested by Murdock (1959). However, if using metric units the standard wake frequency equation has to be used; it also has to be used if a combination of units is used, with the conversion being carried out before calculating f_w.

The following sections deal with the validation of the theoretical models for the calculation of the natural frequency. First, a theoretical background for free and flow-induced vibration is given, section 2.2.1.1. Then, equations for the calculation of the natural frequency are developed (sections 2.2.1.2.1 - 2.2.1.2.4). Using these models a parametric study is carried out to gain an understanding how the natural frequency is influenced when certain thermowell characteristics are changed (section 2.2.1.2.5). Finally, practical vibration analysis is carried out to determine the natural frequency of 'real' thermowells (see section 2.2.1.3 and 2.2.1.4). This information is used to determine which theoretical model is most suitable for the calculation of the natural frequency and should therefore be implemented in the expert system.

2.2.1.1 Theory of Vibration

2.2.1.1.1 Free vibration

Any system that possesses mass and elasticity is capable of free vibration, which is defined as the "vibration that takes place in the absence of external excitation" (Thomson 1993, p.17).

For systems experiencing free vibration the system's natural frequency is of main concern. The natural frequency is characteristic for each individual system, and represents the frequency at which the system will vibrate freely once it has been brought into motion. Small amounts of damping in such systems can be neglected for the calculation of the natural frequency, resulting in a rather conservative system. The only effect damping has in such a case is its influence on the vibration amplitude, which will decrease with time.

The simple oscillatory system in Figure 10 represents the basic vibration model.

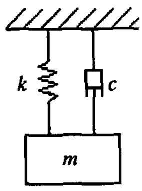


Figure 10: Basic vibration model

In this system, the spring is assumed to be massless and the force-deflection relationship follows Hooke's law: $F = ky$, with k being the spring's stiffness.

The damper can be described by a force-velocity relationship: $F = cv = c\dot{y}$, with c being the damping coefficient.

Assuming that the damping coefficient is sufficiently small, i.e. damping effects can be neglected, and putting the system into motion, oscillation will occur at the natural frequency. An equation for the natural frequency can be established by applying Newton's second law:

$$m\ddot{y} = \Sigma F$$

where m is the mass, $\ddot{y}$ is the acceleration of the mass and ΣF is the sum of all forces acting on the mass.

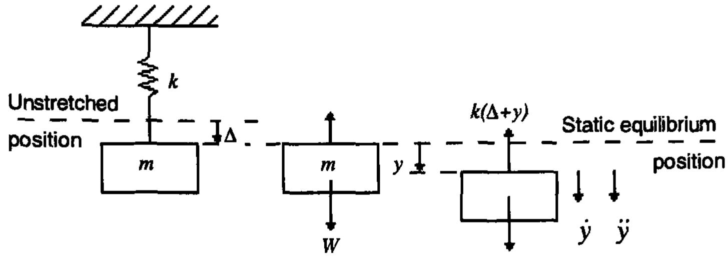


Figure 11: Spring - mass system

Looking at the system in static equilibrium, it can be seen that the mass causes the spring to deform by Δ (see Figure 11). The spring force $k\Delta$ is equal to the gravitational force W acting on the mass:

$$k\Delta = W = mg$$

When the system is put into motion, it will be displaced from its static equilibrium position by y. Therefore, the forces acting on mass m are $k(\Delta+y)$ and W:

$$m\ddot{y} = W - k(\Delta + y)$$
$$m\ddot{y} = -ky$$
$$\ddot{y} + \frac{k}{m}y = 0$$
$$\ddot{y} + \omega_n^2 y = 0$$

where $\omega_n = \sqrt{\frac{k}{m}}$ is the *circular frequency* of the system.

The equation $\ddot{y} + \omega_n^2 y = 0$ describes a system experiencing harmonic motion. Therefore, the general solution of the equation is:

$$y = A\sin\omega_n t + B\sin\omega_n t$$

where A and B are constants that can be established from the initial conditions $y(0)$ and $\dot{y}(0)$.

The natural period of oscillation is

$$\tau_n = 2\pi\omega_n = 2\pi\sqrt{\frac{m}{k}}$$

The natural frequency is the inverse of the natural period, and with $W = mg = k\Delta$ the natural frequency can be written in the form:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}}$$

This is applicable for all systems with a single degree of freedom.

2.2.1.1.2 Flow induced vibration

A bluff body such as a thermowell causes the fluid flow across that structure to separate from the aft contours of the structure. This is due to the insufficiency of the fluid pressure to force the flow about the back of the thermowell. The separation occurs alternately and in a regular manner, causing a vortex street, the so-called 'von Kármán vortices' behind the structure (see Figure 12).

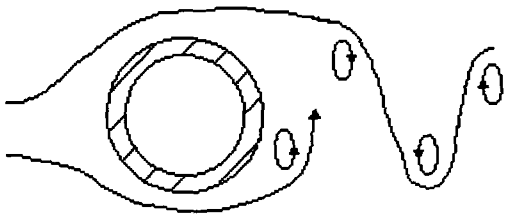


Figure 12: von Kármán vortices

The frequency at which separation occurs can be determined using the dimensionless Strouhal number (Bohl 1991, p174)

$$Sr = \frac{f_w \cdot L}{v}$$

where f_w is the vortex-shedding frequency or wake frequency, L is a characteristic dimension (for cylinders in cross-flow it is the diameter of the cylinder) and v is the velocity of the flow surrounding the structure.

The Strouhal number is a function of the Reynolds number Re. In the range of $10^2 < \mathrm{Re} < 2 \cdot 10^5$, which is defined as the subcritical range (Chen 1987) and is also the typical range of thermowell applications, the Strouhal number can be assumed to be constant at about Sr = 0.22 (Murdock 1959).

Due to the vortex shedding on alternating sides of the cylinder a periodically oscillating force is exerted on the cylinder. The force acts perpendicular to the direction of the fluid flow, i.e. in the cylinder's direction of lift. Written in a form commonly used for aerodynamic forces, the maximum intensity of the force can be expressed as (Den Hartog 1956, p395):

$$F_K = (\frac{1}{2} C_K \rho v^2 A) \sin(2\pi f_w t)$$

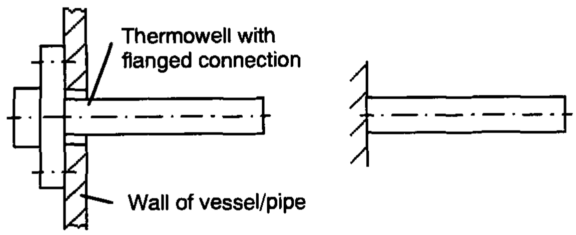
where C_K is the Kármán-force coefficient, ρ is the density of the fluid surrounding the cylinder and A is the projected area of the cylinder exposed to the flow (in cross flow $A=LD$, i.e. length • diameter).

The Kármán-force coefficient is not known with great accuracy but in the same Reynolds number range that is valid for assuming a constant value for Sr, C_k can be set to $C_k=1$.

When the vortex shedding frequency approaches the natural frequency of the cylinder exposed to the flow, a condition called 'lock-in' or 'synchronisation' can occur (Harris 1994). In this case the vibration of the cylinder controls the vortex-shedding, causing the vortex-shedding frequency to lock on to the cylinders natural frequency. The resulting vibrations now occur at the natural frequency of the cylinder, and not at the wake frequency. This can lead to large amplitude structural vibrations, eventually causing damage to the structure.

2.2.1.2 Theoretical Vibration Analysis

The installation of a thermowell, with one end fixed to the process, the other end free and no intermediate support (Figure 13a)) suggests that a thermowell can be modelled as a cantilever beam (Figure 13b)).



a) Thermowell installation **b) Thermowell model**

Figure 13: Modelling of thermowells

There are several methods available to determine the natural frequency of a cantilever beam. Murdock (1959) used the approach discussed in the sections 2.2.1.1.1 *Free Vibration* and 2.2.1.2.1 *Natural frequency of an elastic system with one degree of freedom*, modelling the thermowell as a system with one degree of freedom. Other methods such as Rayleigh's method and Euler equations for beams are discussed briefly in *Appendix II* and applied to the thermowell problem. The results achieved with all these methods will be compared with the experimental results, and the method producing the most accurate results for the natural frequency is implemented in the expert system.

2.2.1.2.1 Natural frequency of an elastic system with one degree of freedom

This is the classical approach for establishing the natural frequency of free vibration of an elastic system with one degree of freedom. It is the same approach Murdock (1959) used for tapered thermowells.

The natural frequency can be calculated from the equation

$$f_n = \frac{1}{2\pi} \cdot \sqrt{\frac{g}{y_{\max}}}$$

where g is the acceleration of gravity (usually taken to be 9.81 ms^{-2}) and $y_{\max}$ is the maximum static deflection of the system (in this case a thermowell) caused by its own weight.

In order to calculate the natural frequency using this equation, an expression for the static deflection has to be derived. As thermowells can be modelled as cantilever beams the basic bending theory of beams can be used. According to this theory, differentiating the deflection of a beam four times results in the load per unit length acting on the beam. If that load is known and the deflection has to be established, integrating the expression for the load four times over the length of the beam will result in an expression for the beam's deflection.

$$EJy'''' = q$$
$$EJy''' = qx + c_1$$
$$EJy'' = \frac{1}{2}qx^2 + c_1x + c_2$$
$$EJy' = \frac{1}{6}qx^3 + \frac{1}{2}c_1x^2 + c_2x + c_3$$
$$EJy = \frac{1}{24}qx^4 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

c_1 to c_4 are constants generated by the integration process; their values depend on the boundary conditions of the beam. E is Young's modulus of elasticity of the beam's material and J is the second moment of area. The product EJ is often referred to as *flexural rigidity* or *bending stiffness*. The second moment of area J depends on the cross-section of the beam; in the case of cylinders it can be calculated using the relationship $J={}^{\pi}/_{64}D^4$ with D being the diameter of the cylinders. For tubes, and therefore for thermowells, $J={}^{\pi}/_{64}(D^4 - d^4)$ with d being the inside diameter and D the outside diameter.

2.2.1.2.2 Parallel thermowells

Applying this theory to parallel thermowells, the load acting on the thermowell has to be established first. As the case of free vibration is assumed, no external forces are acting on the thermowell. Therefore, the only applicable load is the weight of the thermowell. As a parallel thermowell has constant inside and outside diameters along the length (except for the last few millimetres at the free end), it can be represented as a uniformly distributed load (UDL) q_1. The closed end of the thermowell has to be represented by a second UDL q_2 (see Figure 14).

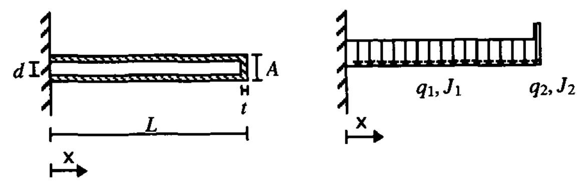


Figure 14: Parallel thermowell model

The UDLs can be calculated by dividing the force caused by the weight by the length of the appropriate thermowell section, i.e.:

$$q_1 = \frac{F_{W1}}{L - t}$$

$$q_2 = \frac{F_{W2}}{t}$$

The forces can be calculated as follows:

$$F_{W1} = W_1 \cdot g$$

$$F_{W2} = W_2 \cdot g$$

with:

$$W_1 = \frac{\pi}{4}(L - t)(A^2 - d^2)G$$

$$W_2 = \frac{\pi}{4} A^2 G t$$

This results in the following expressions for the UDLs:

$$q_1 = \frac{\pi G g(A^2 - d^2)}{4}$$

$$q_2 = \frac{\pi G g A^2}{4}$$

The second moment of area for the two different cross-sections of the thermowell can be calculated using:

$$J_1 = \frac{\pi}{64}\left(A^4 - d^4\right)$$

$$J_2 = \frac{\pi}{64}A^4$$

As can be seen from Figure 14, the load over the length of the thermowell changes at $x=L-t$ from q_1 to q_2. In order to represent this step change and still enable integration, half range functions (Ross 1987, pp158-159) were introduced. This approach is also referred to as *Macauley's Method*. Therefore, the load over the length of the thermowell can be written in the following form:

$$q(x) = q_1(1-[x-(L-t)]^0) + q_2[x-(L-t)]^0$$

Similarly, the second moment of area is:

$$J(x) = J_1(1-[x-(L-t)]^0) + J_2[x-(L-t)]^0$$

It was assumed that the effect of the closed end can be neglected, therefore using a constant J over the length, because t is small compared to the immersion length L. t is in the range of 2-5mm. To confirm this assumption, f_n was calculated using the approach discussed in this section and the approach used for reduced parallel thermowells, the *Moment-Area Method* (see section 2.2.1.2.4 *Reduced Parallel Thermowells*). As suggested, the value for f_n was equal for both methods. It was also established that a difference between the results occurs for thermowells with t>50mm for a length L=135mm, i.e. t>37%L (see Appendix III), which is an unrealistic value for any thermowell. Hence, an equation can be derived that considers $q(x)$ and uses a constant second moment of area $J=J_1$:

$$EJy'''' = q_1(1-[x-(L-t)]^0) + q_2[x-(L-t)]^0$$

Integrating the equation four times and taking the boundary conditions

$$y'(x=0) = 0, y(x=0) = 0, y'''(x=L) = 0, y''(x=L) = 0$$

into consideration results in the equation

$$EJy(x) = \frac{q_1}{24}(x^4 - [x-(L-t)]^4) + \frac{q_2}{24}[x-(L-t)]^4 + \frac{1}{6}(q_1(t-L) - q_2 t)x^3 + \ldots$$

$$\ldots + \frac{1}{2}(\frac{q_1}{2}L^2 + \frac{t^2}{2}(q_1 - q_2) + Lt(q_2 - q_1))x^2$$

Note that for $0 < x \leq L\text{-}t$ the function $[x\text{-}(L\text{-}t)]^4$ is zero (half-range function).

In order to calculate the natural frequency of a parallel thermowell, the maximum deflection is required. Therefore, the equation is reduced to the form:

$$EJy_{\max} = EJy(x = L) = \frac{q_1 L^4}{8} + (\frac{t^4}{24} + \frac{tL^3}{3})(q_2 - q_1) + \frac{t^2 L^2}{4}(q_1 - q_2)$$

If the material of the thermowell is known, E can be established and the equation is divided by EJ, resulting in a value for $y_{\max}$:

$$y_{\max} = \frac{1}{EJ}\left\langle \frac{q_1 L^4}{8} + (\frac{t^4}{24} + \frac{tL^3}{3})(q_2 - q_1) + \frac{t^2 L^2}{4}(q_1 - q_2) \right\rangle$$

This value can then be used to calculate the theoretical natural frequency of the thermowell, i.e.

$$f_n = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{g}{y_{\max}}}$$

Note: Setting q_2=0 and t=0 (i.e. representing an 'open-end thermowell') shows that the equation for $EJy_{\max}$ is the same as can be found in textbooks concerning the displacement equation for cantilever beams with a uniformly distributed load. However, it is not possible to draw any conclusions how the closed end will affect the equation found in the literature. It was therefore necessary to carry out the derivation for this special case.

2.2.1.2.3 Tapered thermowells

For the case of tapered thermowells, the already available equation established by Murdock (1959) was adopted without any change; the process of deriving the equations was checked for any errors, however.

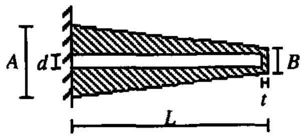


Figure 15: Tapered thermowell model

The equation for calculating the static deflection of a tapered thermowell caused by its own weight therefore is:

$$
\begin{aligned}
y := \frac{4 \cdot G g}{3 \cdot E\left(\frac{A-B}{L}\right)^{4}} \cdot \Bigg[& \frac{1}{2} \cdot (A-B)^{2} \ldots \\
& + \Bigg[\frac{1}{4} \cdot \left[3 \cdot \frac{B^{4}}{d^{2}} - 5 \cdot d^{2} - 6 \cdot \left[B + t \cdot \left(\frac{A-B}{L}\right)\right]^{2}\right] \cdot \left[\frac{B}{d} \cdot \ln\left[\frac{(B-d) \cdot (A+d)}{(B+d) \cdot (A-d)}\right] + \ln\left(\frac{A^{2}-d^{2}}{B^{2}-d^{2}}\right)\right] \ldots \\
& + \left[\frac{1}{2} \cdot \left[6 \cdot \left(B + t \cdot \frac{A-B}{L}\right)^{2} - 7 \cdot d^{2} - \frac{3 \cdot B^{4}}{d^{2}}\right] \cdot \left[\left[\frac{B}{d} \cdot \operatorname{atan}\left[\frac{d \cdot (B-A)}{d^{2}+A \cdot B}\right]\right] + \frac{1}{2} \cdot \ln\left(\frac{A^{2}+d^{2}}{B^{2}+d^{2}}\right)\right]\right] \ldots \\
& + 2 \cdot \left[\frac{B^{3}}{d^{2}} - 3 \cdot \left[B + t \cdot \left(\frac{A-B}{L}\right)\right]\right] \cdot \Bigg[\frac{B}{2} \cdot \ln\left[\frac{\left(B^{2}+d^{2}\right) \cdot \left(A^{2}-d^{2}\right)}{\left(B^{2}-d^{2}\right) \cdot \left(A^{2}+d^{2}\right)}\right] + \Bigg[d \cdot \operatorname{atan}\left[\frac{d \cdot (B-A)}{d^{2}+A \cdot B}\right] \ldots \\
& \qquad + \frac{d}{2} \cdot \ln\left[\frac{(B-d) \cdot (A+d)}{(B+d) \cdot (A-d)}\right]\Bigg]\Bigg]\Bigg]\Bigg]
\end{aligned}
$$

Murdock states that this equation requires the use of seven-place log and trig tables and that the use of the equation in this form is impracticable for routine calculations. As the equation will be calculated using a personal computer these restrictions do not apply.

Table 3: Comparsion between calculation methods

Element size / Immersion length	¼"	$^{3}/_{8}$"	$^{9}/_{16}$"	$^{11}/_{16}$"	$^{7}/_{8}$"
2.5"	-0.12	-0.01	0.33	0.08	-0.13
4.5"	-0.32	0.06	-0.23	-0.22	-0.15
7.5"	-0.20	-0.16	0.07	-0.05	-0.09
10.5"	0.12	-0.02	-0.06	-0.05	-0.11
16"	-0.01	-0.25	-0.13	-0.25	-0.29
24"	-0.08	-0.43	-0.38	-0.30	-0.47

A comparison of the natural frequency calculated using the above equation and the K_f-factors given in the PTC is shown in Table 3; the values represent the difference between the natural frequencies in percent of the frequency calculated using y_{max}.

As expected, both ways of establishing f_n give the same results. The slight deviations can be explained by rounding errors when the K_f-factors were first established.

2.2.1.2.4 Reduced parallel thermowells

In order to calculate the natural frequency of a reduced parallel thermowell using the same equation as for parallel and tapered thermowells, an equation for establishing the static deflection of the well has to be derived. An approach especially useful for beams with a step variation in the sectional properties is the Moment-Area Method (Ross 1987, pp166-168). The method is explained with the example of a cantilever beam with an end-load:

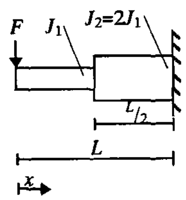


Figure 16: Cantilever with step change in cross-section

First, the bending moment caused by the weight of the beam has to be established:

$$M(x) = F \cdot x$$

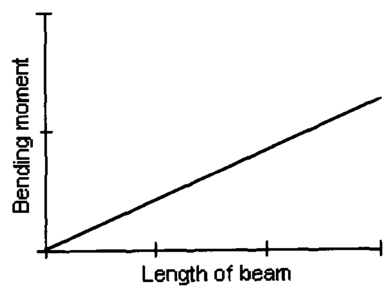


Figure 17: Bending moment of reduced beam

Then, the bending moment has to be divided by the second moment of area J:

$$\frac{M}{J}(x) = \frac{M(x)}{J(x)}$$

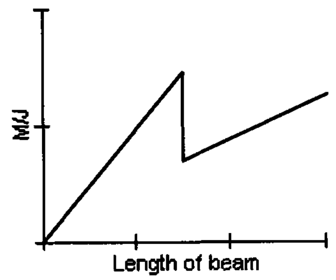


Figure 18: $^{M}/_{J}$ diagram of reduced beam

It is argued that the deflection of the beam can now be established by multiplying the area of the $^{M}/_{J}$ diagram by its centre of gravity, x_G and then dividing it by E.

$$y = \frac{1}{E} \cdot Area_{M/_{J(x)}} \cdot x_G$$

In this example, it was assumed that the load is a point-load at the end of the beam, and that J_2=2J_1 and L_1=L_2=$^{L}/_{2}$. This results in a linear moment-area and straightforward determination of x_G. In the case of thermowells, however, these assumption cannot be made. As was discussed for parallel thermowells, a uniformly distributed load to represent the weight of the thermowell should be used, resulting in a non-linear moment-area diagram (Figure 19).

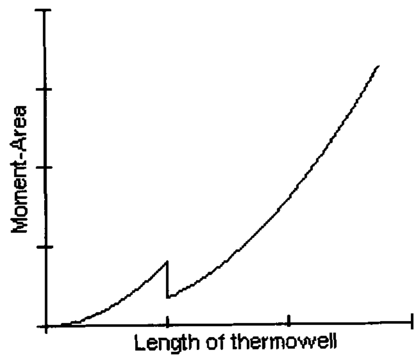
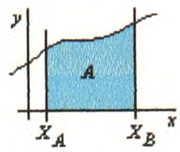


Figure 19: Moment-area of reduced parallel thermowell

Also, for thermowells the relationships between the larger and smaller diameters (and therefore between the J of each section), or between the standard and reduced lengths will vary for each individual application.

The non-linear shape of the $^{M}/_{J}$-curve makes the determination of the area under the curve and the centre of gravity more complicated than for the linear case. An equation was found that allows the calculation of the centre of gravity of the area under a curve between two boundaries x_A and x_B:

$$x_G = \frac{\int_{x_A}^{x_B} xy\,dx}{\int_{x_A}^{x_B} y\,dx} = \frac{M_y}{A}$$



where A is the area under the curve, which in this particular case is $A = Area_{M/J(x)}$ and M_y is the *statische Moment einer Fläche* (static moment of an area) (Bartsch 1990, pp436-437). The boundaries are $x_A = 0$ (the free end) and $x_B = L$ (the fixed end).

Rearranging the previously established equation for the deflection and the equation for x_G:

$$M_y = A \cdot x_G = Area_{M/J(x)} \cdot x_G$$

$$E \cdot y = Area_{M/J(x)} \cdot x_G$$

$$\therefore M_y = E \cdot y$$

Therefore, the deflection of a reduced parallel thermowell can be calculated using the relationship:

$$y = \frac{M_y}{E} = \frac{1}{E} \cdot \int_0^L x \frac{M(x)}{J(x)} dx$$

Figure 20 illustrates the relevant dimensions used in the equations. The left hand side shows the thermowell geometry, the right hand side shows the changes in the distributed load and the second moment of area of the thermowell.

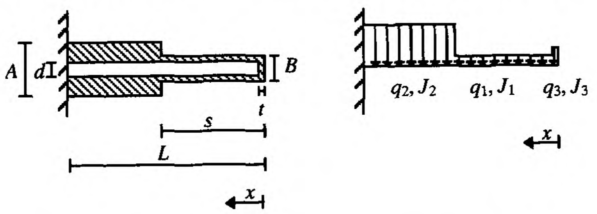


Figure 20: Reduced parallel thermowell model

Using Macauley's method, equations for $q(x)$, $M(x)$ and $J(x)$ can be established:

$$EJy''''(x) = q(x) = q_3(1 - [x-t]^0) + q_1([x-t]^0 - [x-s]^0) + q_2[x-s]^0$$

$$EJy''(x) = M(x) = \frac{1}{2}q_3(x^2 - [x-t]^2) + \frac{1}{2}q_1([x-t]^2 - [x-s]^2) + \frac{1}{2}q_2[x-s]^2$$

$$J(x) = J_3(1 - [x-t]^0) + J_1([x-t]^0 - [x-s]^0) + J_2[x-s]^0$$

Using these equations, the maximum deflection of the thermowell can be calculated. However, there is no symbolic solution for the integral (the static moment), therefore integration has to be carried out numerically for each individual thermowell. But this can easily be done using a computer or a programmable calculator.

2.2.1.2.5 Parametric study

To gain an understanding of how the natural frequency f_n of a thermowell changes when certain characteristics such as the root diameter A or the immersion length L are modified, a parametric study was carried out. The results of this study were also used to decide on the thermowell geometry for the practical vibration analysis.

The deflection method was used to calculate the natural frequency for both parallel and tapered thermowells.

The parametric study was carried out without paying too much attention to the practical limitations of thermowell manufacture. For example, tapered thermowells can only be manufactured up to a length of 400mm. It is not possible to drill a bore deeper than 400mm with the available gun drill. The outside diameter, on the other hand, is restricted by the process connection, the tube size or the maximum diameter of the solid bar used. These restrictions, however, will not affect how the natural frequency behaves when dimensions are changed and they could therefore be neglected in the parametric study.

<u>- parallel thermowells</u>

The first investigations were carried out using a parallel thermowell model. Figure 21 shows the behaviour of the natural frequency when increasing the thermowell's length for a constant outside diameter and a constant bore diameter. The variation of the length was carried out for different outside diameters, but with a constant bore diameter for all models. The graph indicates that the natural frequency decreases logarithmically with increasing length and increases with increasing outside diameter. The difference in natural frequency for all outside diameters is small at lengths larger than 500mm.

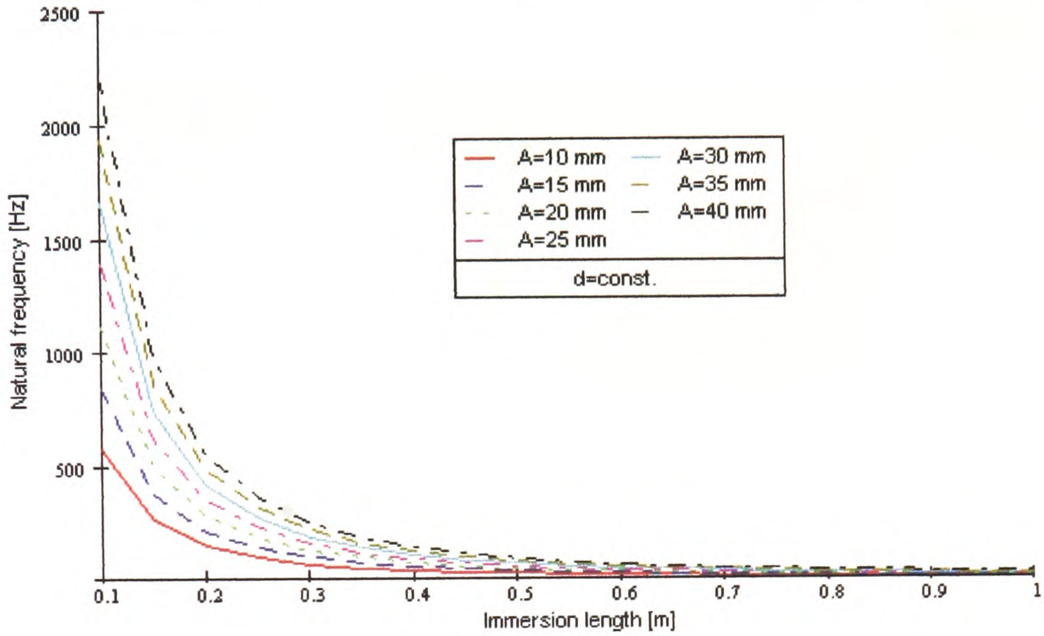


Figure 21: Natural frequency of parallel thermowells for varying length and diameter

In Figure 22 the outside diameter of a thermowell has been increased for each constant length, again with the bore diameter being constant throughout. The natural frequency increases linearly with increasing outside diameter and decreasing length.

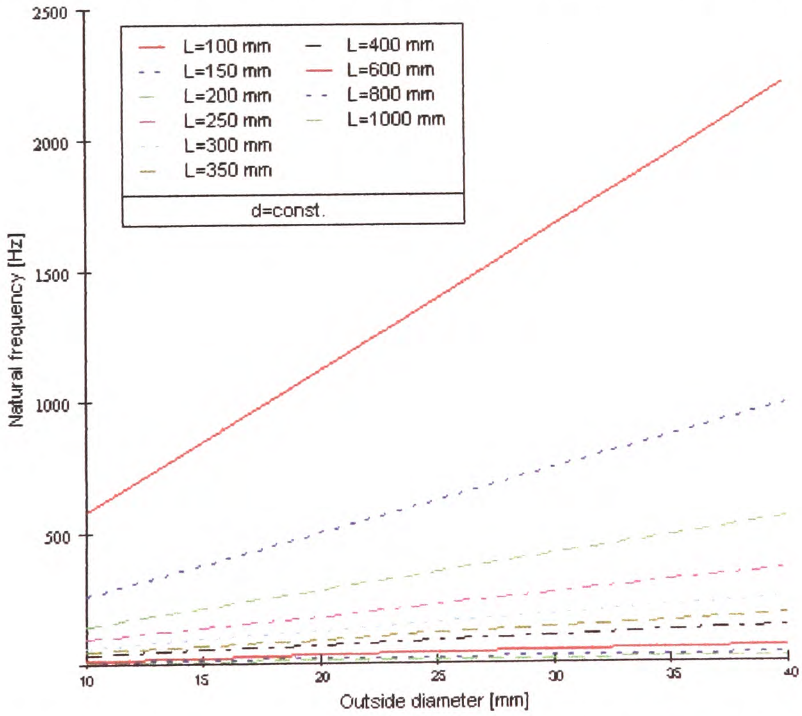


Figure 22: Natural frequency of parallel thermowells for varying diameter and length

Figure 23 combines the previous two results in a three-dimensional representation.

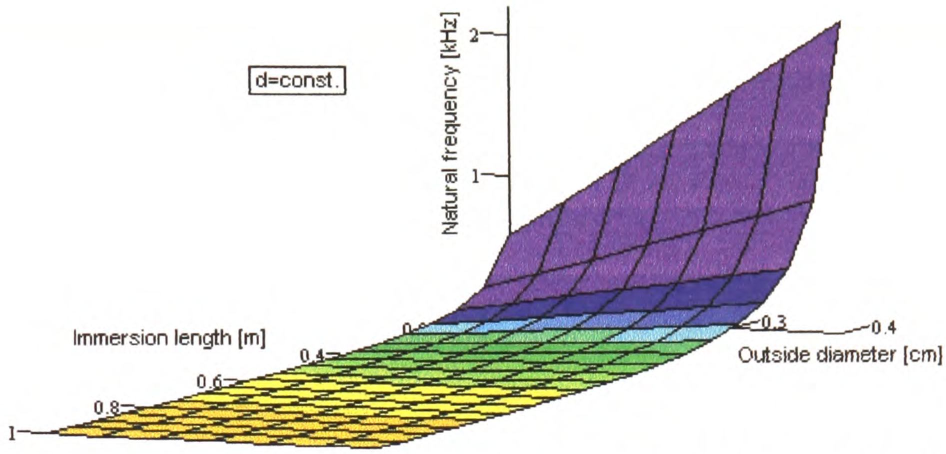


Figure 23: Natural frequency of parallel thermowells for varying length and diameter

The observations made can be explained as follows:

The natural frequency of short thermowells is higher than for long thermowells for two reasons. First, the longer the thermowell, the higher the weight; this in turn causes a larger deflection. Second, the deflection itself depends on the length of the thermowell, increasing with increasing length. The overall higher deflection of long thermowells results in a lower natural frequency because the natural frequency is indirectly proportional to the deflection, i.e. $f_n \sim {}^1/_y$.

The natural frequency increases with increasing outside diameter because the deflection decreases. Even though the weight of the thermowell increases with the outside diameter (the bore diameter is constant, therefore the wall thickness increases adding more material), the flexural rigidity increases even more ($EJ \sim A^4$ whilst weight $q \sim A^2$), thus decreasing the deflection because $y \sim {}^q/_{EJ}$ and therefore increasing the natural frequency.

Another study was carried out to investigate the effect of varying bore diameters. A representative graph is shown in Figure 24 for one thermowell length and increasing outside diameter. Four cases were assumed:

- the bore diameter stays constant, therefore the wall thickness increases (this is the same case as investigated previously; compare with Figure 22, L=100mm)

- the bore diameter is increased by the same value as the outside diameter, resulting in a constant wall thickness
- the bore diameter is increased by less than the outside diameter; therefore the wall thickness increases, but by less than for a constant bore diameter
- the bore diameter is increased by more than the outside diameter thus decreasing the wall thickness

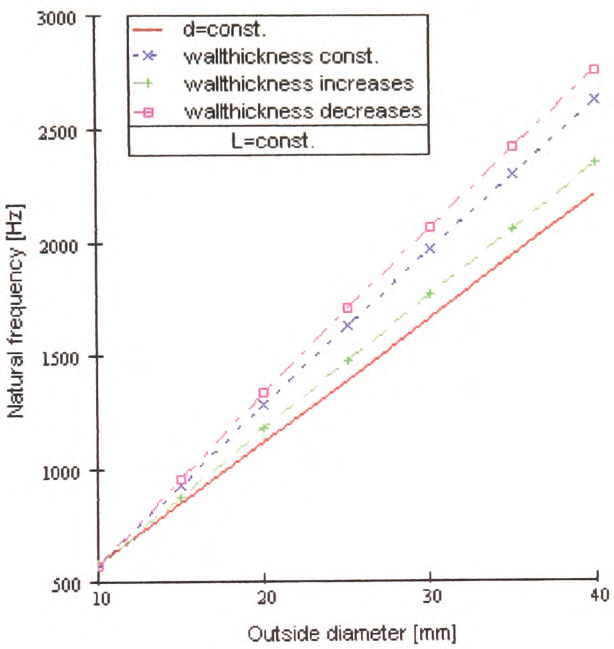


Figure 24: Natural frequency of parallel thermowells for changing outside and bore diameter

From Figure 24 it can be seen that the lowest natural frequency is achieved by keeping the bore diameter constant when increasing the outside diameter. The highest natural frequency is achieved by increasing the bore diameter more than the outside diameter. However, the latter can only be done to a certain extent, because a wall thickness of zero or less is not possible and a specific wall thickness has to be used to withstand the pressure in a practical application. Increasing the bore diameter proportionally to the outside diameter or less will increase the natural frequency compared to keeping the bore diameter constant. This trend is also true for other thermowell lengths.

- tapered thermowells

The first investigations were carried out for thermowells with a constant taper throughout each individual series, i.e. when increasing the length. At the same time, the tip and bore diameter are kept at the same values for all series. This requires that the root diameter be increased each time the length is increased, see Figure 25. Therefore, if the length of the thermowell is increased n times, the root diameter has to be increased n times, too. The curves in Figure 26 representing the change in natural frequency with change in length are identified by the original root diameter A_0 of each series. The graph indicates a similar behaviour of the natural frequency as was observed for parallel thermowells. However, the difference in f_n for different outside diameters at lengths larger than 500mm is greater than for parallel thermowells.

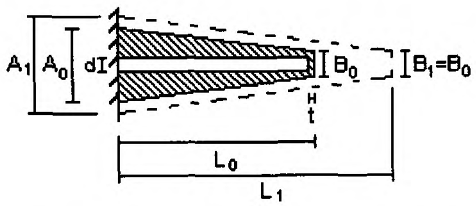


Figure 25: Tapered thermowells with constant taper

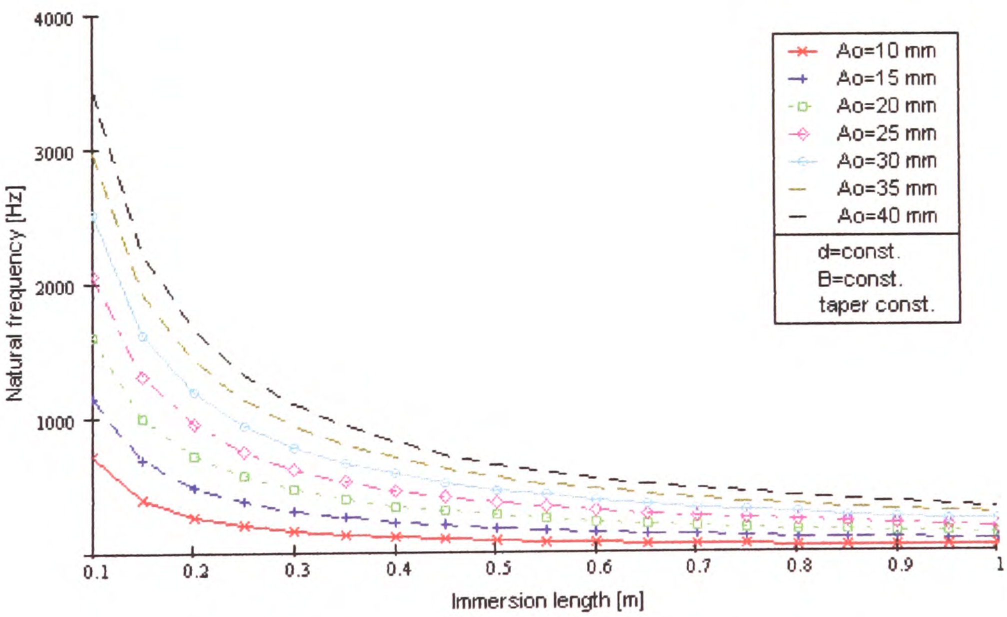


Figure 26: Natural frequency of tapered thermowells for varying length and root diameter

Figure 27 shows the relationship between natural frequency and root diameter, similar to Figure 22 for parallel thermowells. However, because the root diameter has to be increased with each increase of *L*, the root diameters for which the natural frequency has been calculated will be different for each *L* series.

Figure 27 also indicates that keeping a constant taper when increasing the thermowell length can result in an impractical root diameter. The maximum root diameter calculated in this theoretical study is 346mm (for *L*=1000mm), which is not possible in practice.

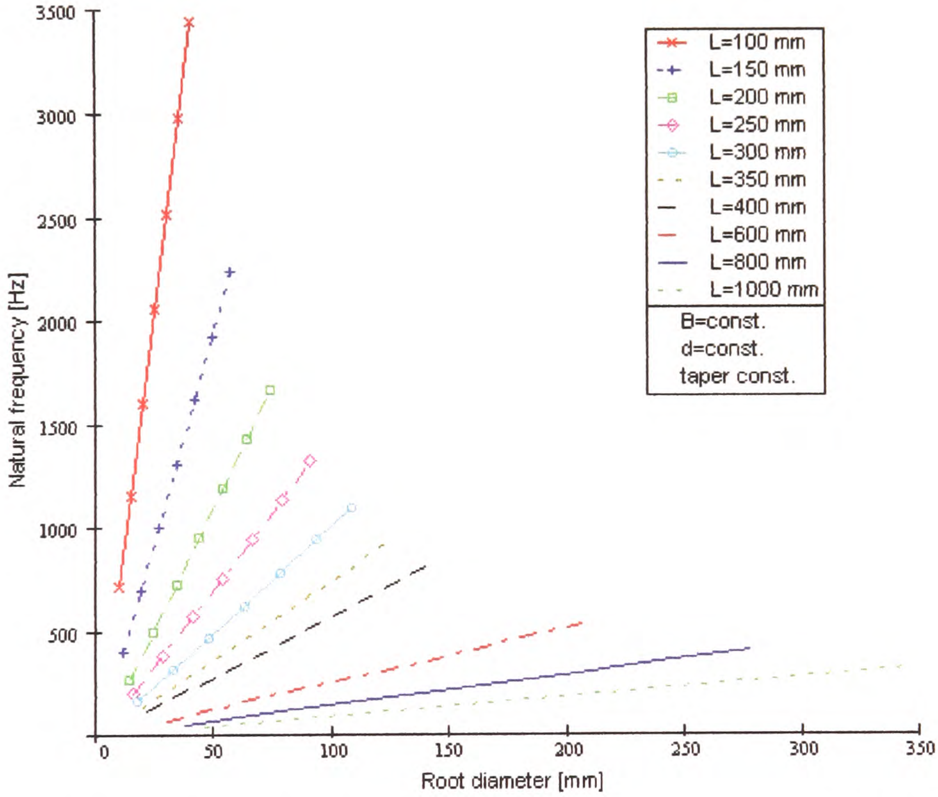


Figure 27: Natural frequency of tapered thermowells with varying root diameter and length

Figure 28 combines the previous two results in a three-dimensional representation.

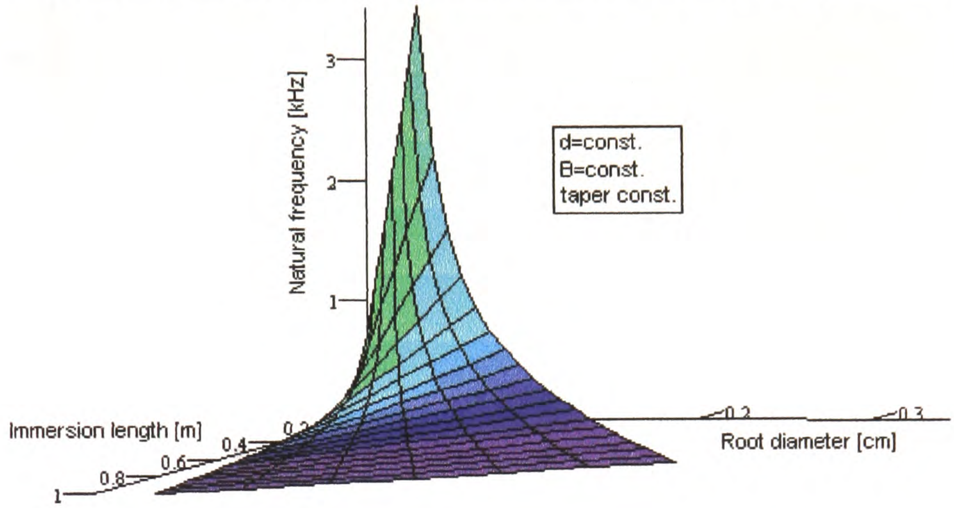


Figure 28: Natural frequency of tapered thermowells for varying length and root diameter

The behaviour of the natural frequency of tapered thermowells with constant root and tip diameter throughout each series of increasing length was also investigated (Figure 29).

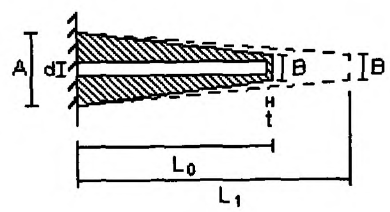


Figure 29: Tapered thermowells with constant root and tip diameter

Figure 30 shows the relationship between the natural frequency and the immersion length of the thermowell for a constant root diameter A and tip diameter B. The root and tip diameters are both increased by 5mm for each new series. The graph indicates a similar characteristic as was seen in Figure 21 for parallel thermowells, with only a small difference between the natural frequency of each thermowell geometry at lengths lager than 500mm.

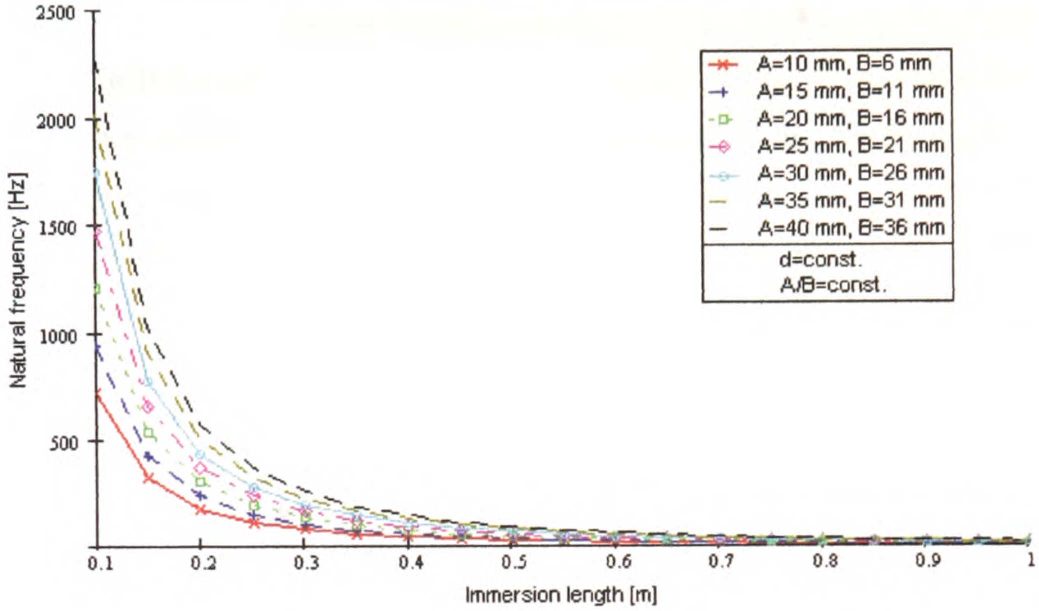


Figure 30: Natural frequency of tapered thermowells for varying length

The trend observed in Figure 31 is also similar to what has been observed for parallel thermowells (Figure 22): increasing the outside diameters at a fixed length increases the natural frequency linearly.

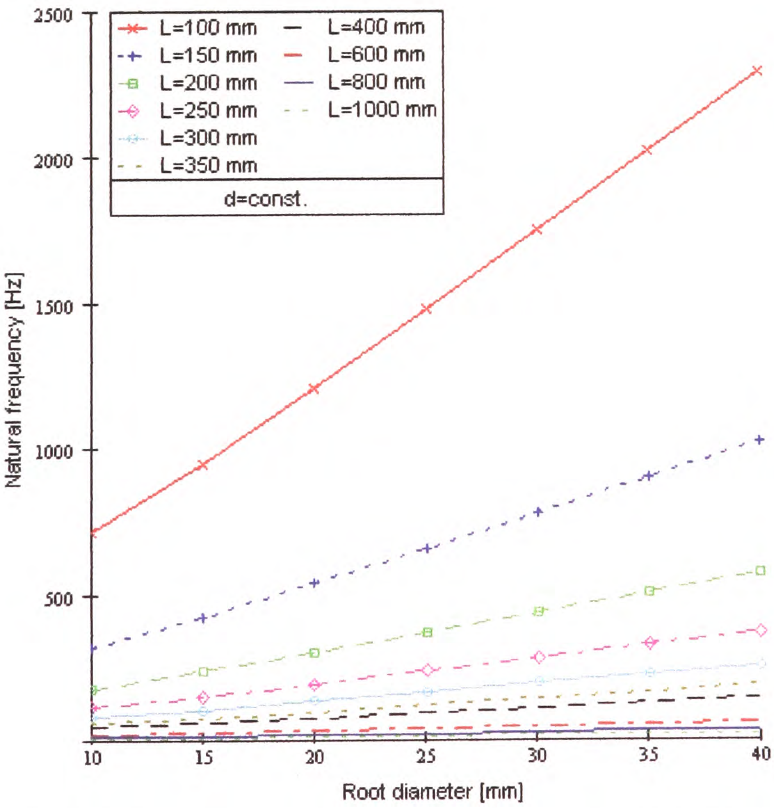


Figure 31: Natural frequency for tapered thermowells for varying root diameter

The reason for this similarity is that in this second tapered thermowell case only the length will be increased and no changes are made to the outside diameters. Therefore no changes to the flexural rigidity are made. If the root diameter is increased, too, the flexural rigidity is increased accordingly.

If the outside diameters are increased for a constant length, the weight will be increased, together with the flexural rigidity. However, if only the root diameter is increased the weight increase is less than when the tip diameter is increased as well, thus causing a smaller deflection and therefore a higher natural frequency.

A combination of the previous two graphs is given in Figure 32.

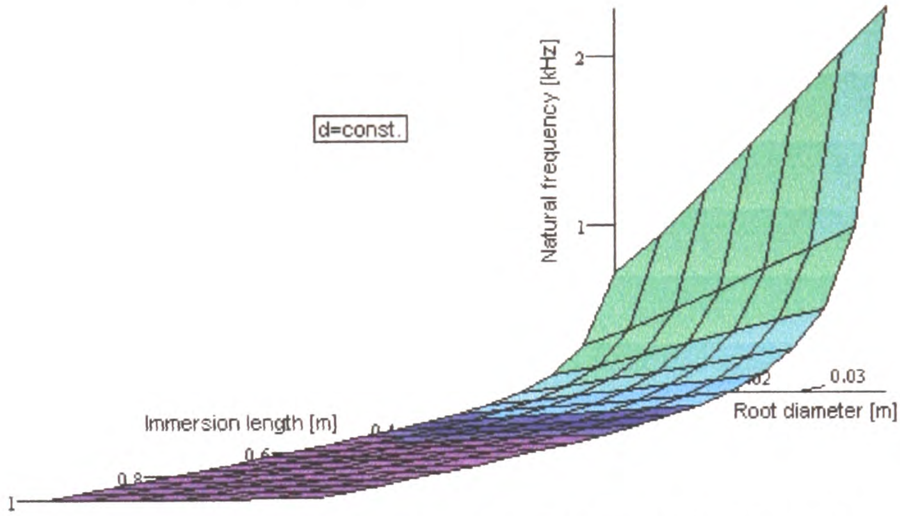


Figure 32: Natural frequency of tapered thermowells for varying length and diameters

<u>- reduced parallel</u>

A somewhat different study was carried out for reduced parallel thermowells. For the design of a reduced parallel thermowell it is important to know how the length of the reduced part influences the thermowell's natural frequency. Therefore, a reduced parallel thermowell was analysed by changing the reduced length and calculating the natural frequency for each new reduced length. The reduced length was varied from 0, thus representing a parallel thermowell with outside diameter A, to the immersion length L, therefore representing a parallel thermowell with outside diameter B.

This was repeated twice, each time increasing diameter A. Then, keeping A at the last (and therefore largest) value, the procedure was repeated again twice, this time increasing the diameter B instead. The MATHCAD sheet for the first thermowell geometry is given in Appendix IV.

The following thermowell geometry was used:

A = 20mm, B = 16mm, d = 8mm, L = 205mm, s = 0 - 205mm

In Figure 33 the natural frequency is plotted against the reduced length for this geometry. Table 4 and Figure 34 highlight the peak values for f_n together with the appropriate reduced length expressed in percent of the immersion length. The results indicate that the maximum natural frequency is reached when the reduced length is 95mm long, i.e. if the diameter is reduced from A to B at 52.2% of the immersion length. However, the graph indicates that there is a 'maximum range' as the difference between the frequencies is minimal.

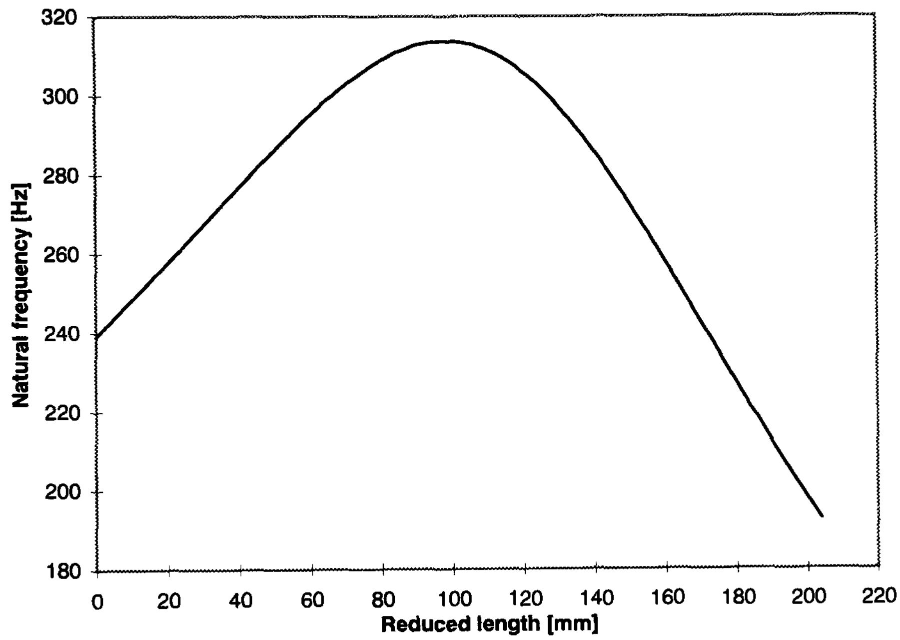


Figure 33: Natural frequency of reduced parallel thermowells with changing reduced length

Table 4: Natural frequency of a reduced parallel thermowells with changing reduced length; peak values

Reduced length [mm]	95	96	97	98	99	100	101	102	103	104
% of immersion length	53.66	53.17	52.68	52.20	51.71	51.22	50.73	50.24	49.76	49.27
Natural frequency [Hz]	313.5	313.6	313.6	313.7	313.6	313.6	313.6	313.5	313.4	313.2

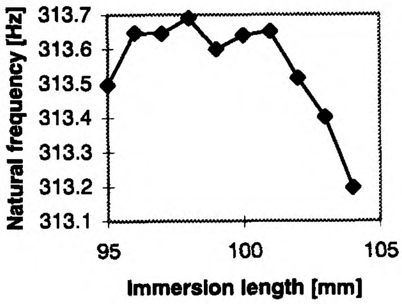


Figure 34: Natural frequency of a reduced parallel thermowells with changing reduced length; peak values

The diameter A was increased by 4mm each time, from 16mm to 24mm. Figure 35 and Figure 36 compare the values of f_n for thermowells with different diameters A and constant diameter B = 12mm.

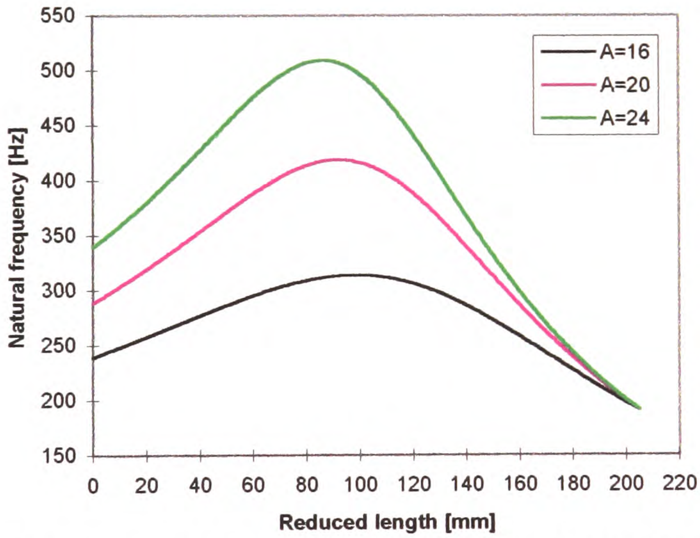


Figure 35: Natural frequency of reduced parallel thermowells with changing reduced length and diameter A

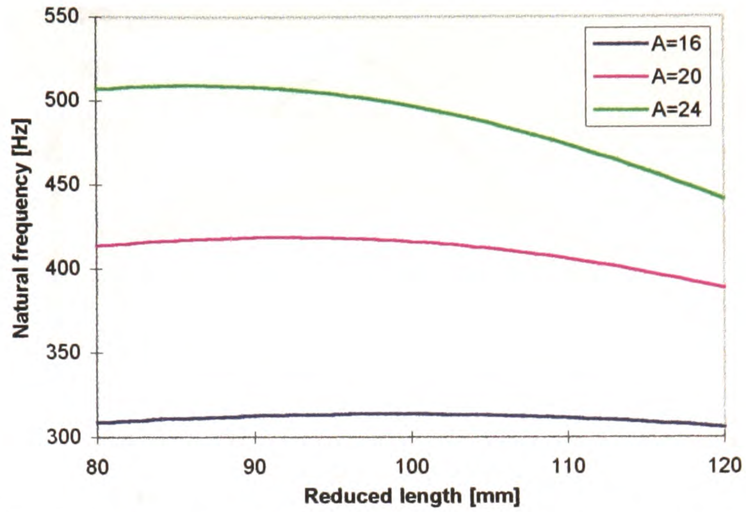


Figure 36: Natural frequency of reduced parallel thermowells with changing reduced length and diameter A; peak values

This study shows that the maximum natural frequency not only depends on the length of the immersion length, but also on the diameter A of the thermowell. With increasing diameter the natural frequency of the thermowell increases (Figure 35). As expected, the natural frequency is equal for all three thermowells if a reduced length of 205mm is used, which represents a parallel thermowell with a constant diameter of 12mm. Also, the diameter A influences for which reduced length a maximum natural frequency is achieved. Figure 36 indicates that the reduced length required to obtain a high natural frequency is decreasing with increasing diameter A.

Next, the diameter B was increased by 4mm each time, from 12mm to 20mm. This was done for a reduced parallel thermowell with a diameter A of 24mm. Figure 37 and Figure 38 compare the values of f_n for thermowells with different diameters B.

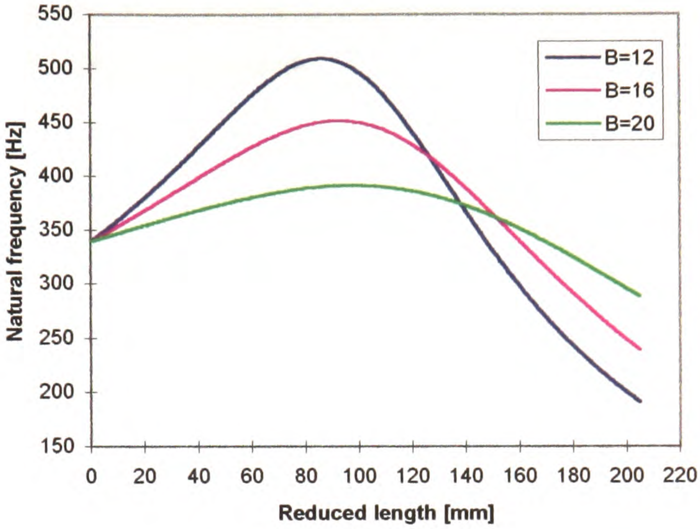


Figure 37: Natural frequency of reduced parallel thermowells with changing reduced length and diameter *B*

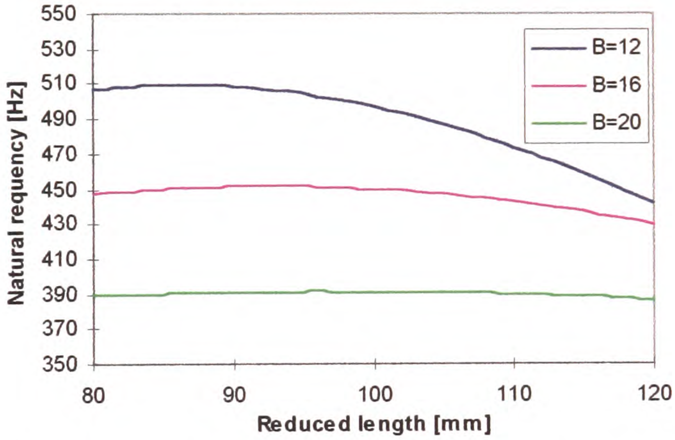


Figure 38: Natural frequency of reduced parallel thermowells with changing reduced length and diameter *B*; peak values

The graphs show that the natural frequency decreases with increasing reduced diameter B for reduced lengths smaller than 120mm. If the reduced length is larger than 160mm, the natural frequency increases with increasing diameter B. Similarly to the observations made for increasing diameter A, the maximum natural frequency depends on the reduced length. In this case, however, it is necessary to increase the reduced length to achieve a maximum frequency if the reduced diameter is increased.

Finally, both diameters A and B were increased by 4mm at the same time. Figure 39 and Figure 40 show the results of this study.

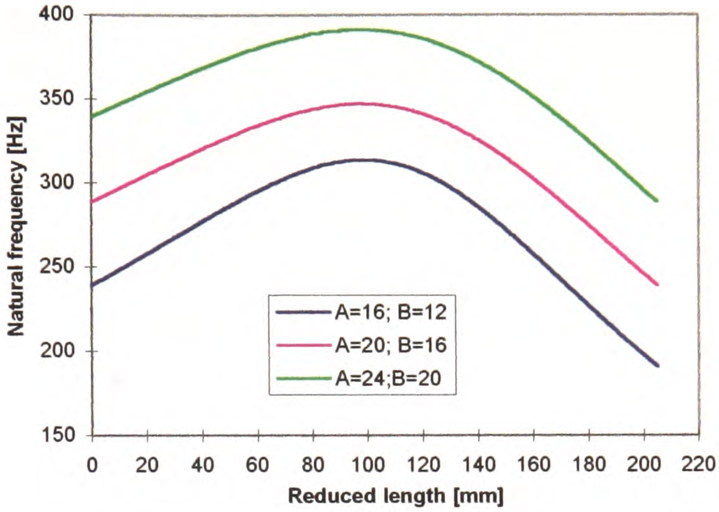


Figure 39: Natural frequency for reduced parallel thermowells with changing reduced length and diameters A and B

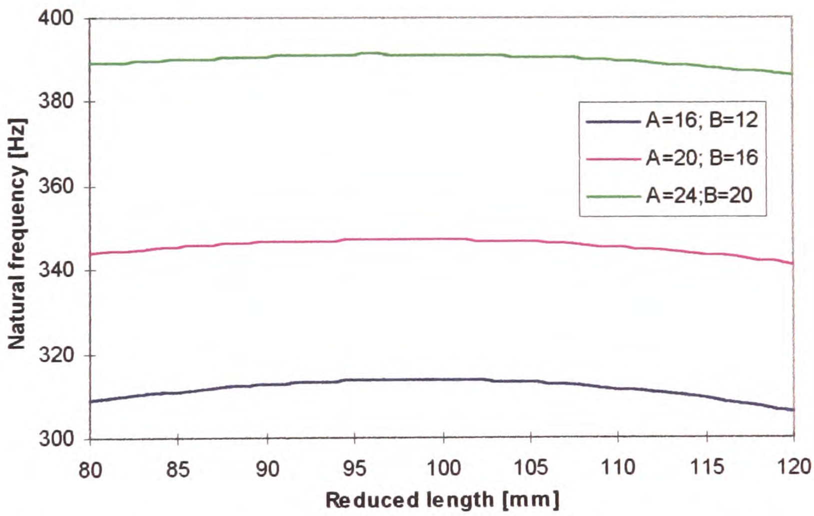


Figure 40: Natural frequency for reduced parallel thermowells with changing reduced length and diameters A and B; peak values

The graphs show that the natural frequency of a reduced parallel thermowell increases with increasing diameters A and B. However, the overall maximum frequency is smaller than the resulting frequency when increasing diameter A only.

All three investigations are combined in Figure 41.

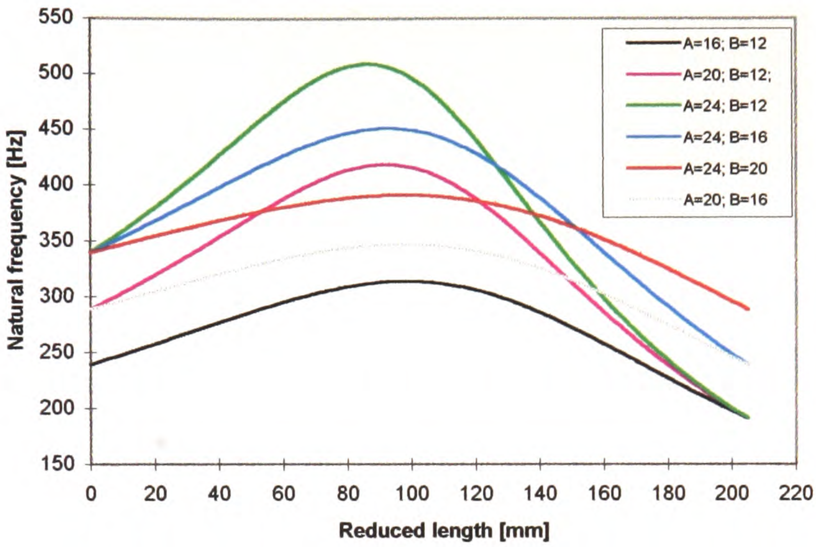


Figure 41: Natural frequency of reduced parallel thermowells

From the investigations carried out, it is clear that the best way of increasing the natural frequency of a reduced parallel thermowell is achieved by increasing the diameter *A* and keeping *B* at the original diameter. In respect to an optimal reduced length it can be seen that the peak of each curve in the previous figures is quite flat, indicating that the difference between individual frequencies in this 'maximum range' is very small (compare also with Figure 36, Figure 38 and Figure 40). As it is impractical to determine the optimal reduced length for each thermowell (it took MATHCAD about 10 minutes to carry out the calculation for each thermowell), it is advisable to set the reduced length to less than 50% of the immersion length. To conclude the study, investigations similar to those for parallel and tapered thermowells were carried out. Figure 42 and Figure 43 show the resulting graphs for reduced parallel thermowells with a reduced length of 46% of the immersion length. These graphs are comparable with the graphs for the tapered and parallel thermowells.

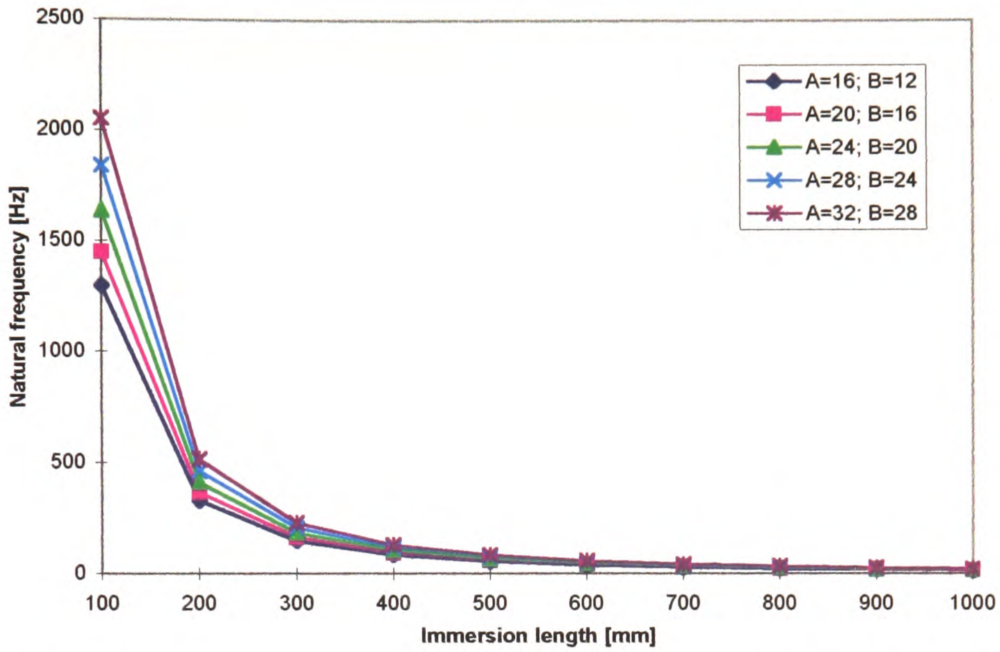


Figure 42: Natural frequency of reduced parallel thermowells for varying length and diameter

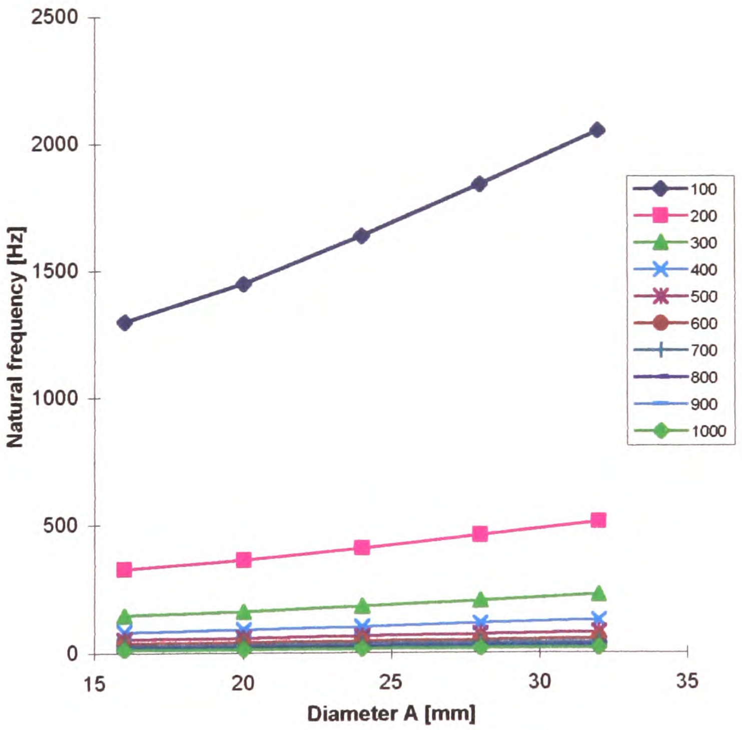


Figure 43: Natural frequency of reduced parallel thermowells for varying diameter and length

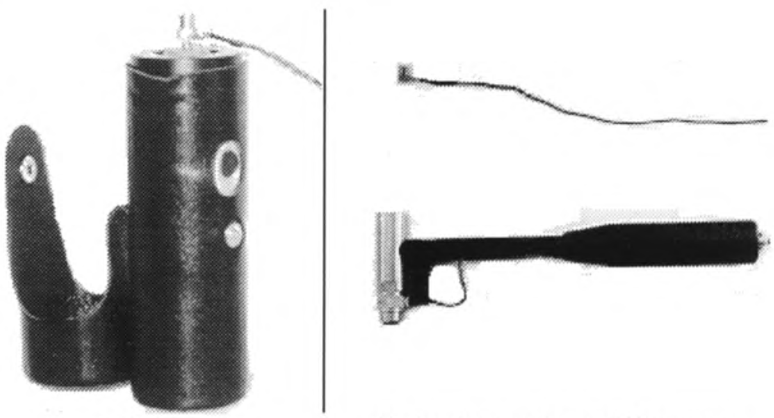
2.2.1.3 *Practical Vibration Analysis*

In order to validate the mathematical models used to determine the natural frequency of the thermowells, the natural frequency of thermowells have to be established using vibration analysis techniques. These 'real' values of f_n can then be compared with theoretical values, allowing a verification of the theoretical approaches.

2.2.1.3.1 Equipment

The following equipment was used for the tests:

- Brüel & Kjær Dual Channel Signal Analyser Type 2032
- Brüel & Kjær Conditioning Amplifier Type 2626
- Brüel & Kjær Calibration Exciter Type 4294 (see Figure 44 a))
- Brüel & Kjær Accelerometer Type 2313 (see Figure 44 a))
- Brüel & Kjær Impact Hammer Type 8202 (see Figure 44 b))
- Viglen 486 DX33 personal computer
- STAR software
- vice (for preliminary tests)
- test rig (for the actual validation tests)



a) Exciter b) Accelerometer and Hammer

Figure 44: Testing equipment

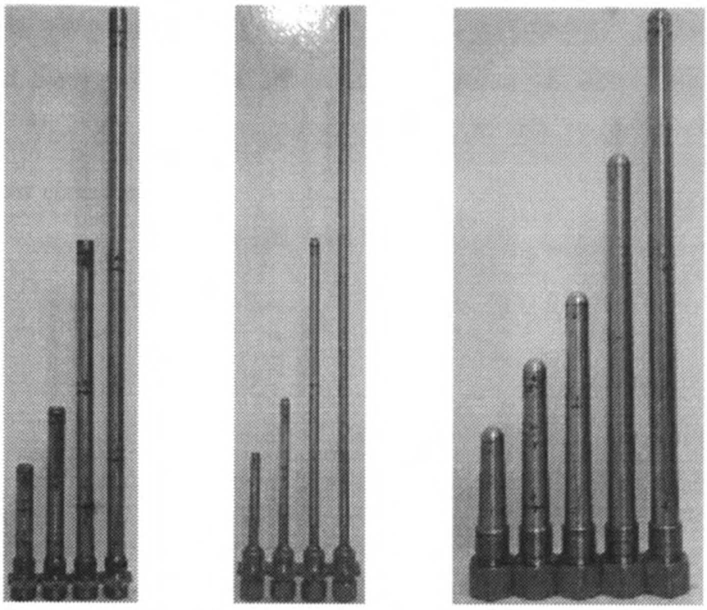
It was decided to test both parallel fabricated thermowells and tapered thermowells machined from solid barstock in order to validate the theoretical approaches for calculating f_n. Taking the parametric study into consideration, the investigation was restricted to thermowell lengths between 100mm and 500mm because the change in natural frequency is more predominant in this area. To confirm the trend that the natural frequency changes with the wall thickness and outside diameter, thermowells with two different diameters were used. However, this investigation was restricted to parallel thermowells because they are less expensive and time-consuming to manufacture and this aspect could not be neglected. Therefore, the thermowell geometries in Table 5 and Table 6 were chosen; Figure 45 shows the appropriate thermowells.

Table 5: Geometries of fabricated parallel thermowells

Nominal length [mm]	Outside diameter [mm]	Bore diameter [mm]	Immersion length [mm]	End thickness [mm]
100	16	13.1	84	2.8
150	16	13.1	135	2.8
300	16	13.1	287	2.8
500	16	13.1	486	2.8
100	10.5	6.5	88	4
150	10.5	6.5	139	4
300	10.5	6.5	287	4
500	10.5	6.5	488	4

Table 6: Geometries of solid tapered thermowells

Nominal length [mm]	Root diameter [mm]	Tip diameter [mm]	Bore diameter [mm]	Immersion length [mm]	End thickness [mm]
100	22	18	8.2	75	8
150	22	18	8.2	125	8
200	22	18	8.2	175	8
300	22	18	8.2	275	8
400	22	18	8.2	375	8



a) first set of parallel thermowells b) second set of parallel thermowells c) tapered thermowells

Figure 45: Thermowells for tests

2.2.1.3.2 Preliminary considerations

Murdock (1959) carried out tests on tapered thermowells; Blevins *et al.* (1996) used tapered and parallel thermowells for their experiments. Murdock (1959) used a vice to fix the thermowells; the arrangement used by Blevins *et al.* (1996) to fix the thermowells consisted of a nozzle and flange, simulating a typical fixing used in an application. When using a vice the clamping force to hold the thermowell in place will only be applied in one direction. It is therefore possible to move the thermowell in the other directions, provided an appropriate force is applied. Therefore, the case of a cantilever beam is not simulated and differences between theoretical and practical vibration analysis have to be expected. On the other hand, when connecting a flanged thermowell with a mating flange the thermowell can be assumed to be a cantilever. However, the flange/nozzle combination used by Blevins *et al.* (1996) can introduce a different error, because the nozzle will vibrate with the thermowell when excited.

Because the use of a vice is a cost-effective solution to hold a thermowell in place it was used in preliminary tests to become familiar with the vibration analysis equipment. These tests were also used to determine whether a vice is sufficient for the tests or if a specifically designed rig has to be used.

Two sets of tests were carried out initially. The first tests were carried out using only the fabricated thermowell with an outside diameter of 16mm and a nominal length of 150mm. The thermowell was clamped with the jaws of a vice at the hexagonal connector (see Figure 46).

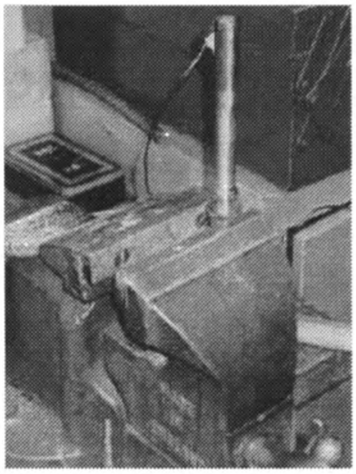
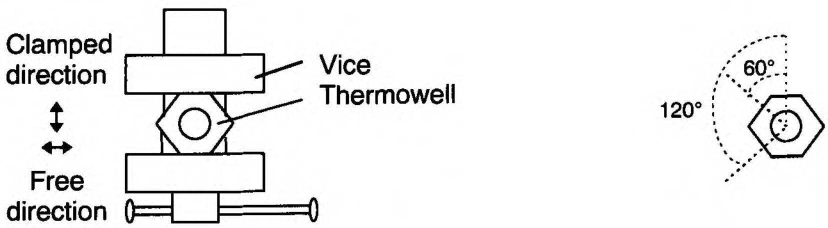


Figure 46: Thermowell clamped in vice; accelerometer attached in 'clamped' direction

The thermowell was then excited in the direction of the clamped side, i.e. in direction of the jaws; the response was measured in the same direction. This is illustrated in Figure 46. Then the thermowell was excited in the free direction (i.e. perpendicular to the clamped direction); the response was also measured in that direction. See Figure 47 a) for an illustration of the terms clamped and free direction.



a) Clamped and 'free' direction **b) Rotation of thermowell**

Figure 47: Thermowell clamped in vice

This procedure was repeated twice after rotating the thermowell by 60° each time to clamp the thermowell at different sides of the hexagonal connector (Figure 47 b)).

The results of this investigation can be seen in Table 7.

Table 7: Results of vibration analysis of a thermowell clamped in a vice

Clamping direction	0°	60°	120°
Direction of excitation and measurement	Natural frequency [Hz]		
Clamped direction	517	498	530
'Free' direction	432	413	453
Difference in frequency	85	85	77

There is a difference of about 80 Hz when determining the natural frequency by exciting/measuring in the clamped or free direction, with the frequency being lower in the free direction. The difference in natural frequency when clamping the thermowell at different sides of the connector is caused by different forces applied when clamping the thermowell. Both cases show that the method of clamping the thermowell influences the measured natural frequency of the thermowell.

The second part of the preliminary tests was carried out with all four available parallel thermowells with the outside diameter of 16mm. Excitation of the thermowell and measurement of the response was carried out in the clamped direction. Table 8 shows the results of these tests and also the values of the natural frequency determined with different theoretical approaches.

Figure 48 and Figure 49 represent the results in graphical form.

Table 8: Comparison between experimental results and various theoretical approaches to establish the natural frequency of parallel thermowells; clamped in a vice

Nominal length [mm]	Immersion length [mm]	Vice [Hz]	Rayleigh [Hz]	Blevins [Hz]	Moment-Area [Hz]	Deflection [Hz]
100	84	662.00	2063.40	2055.20	1653.30	1524.00
150	135	418.00	798.90	795.70	640.10	607.00
300	287	110.00	176.80	176.10	141.60	138.00
500	486	45.00	61.60	61.40	49.40	49.00
		Deviation from test results [%]				
			211.69	210.45	149.74	130.21
			91.12	90.36	53.13	45.22
			60.73	60.09	28.73	25.45
			36.89	36.44	9.78	8.89

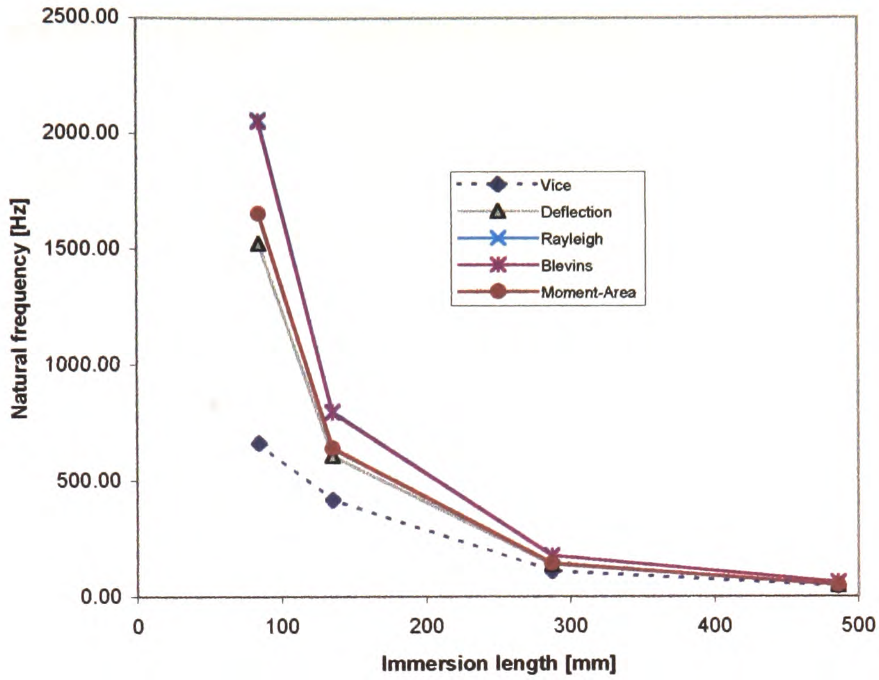


Figure 48: Natural frequency of fabricated parallel thermowells; clamped in a vice

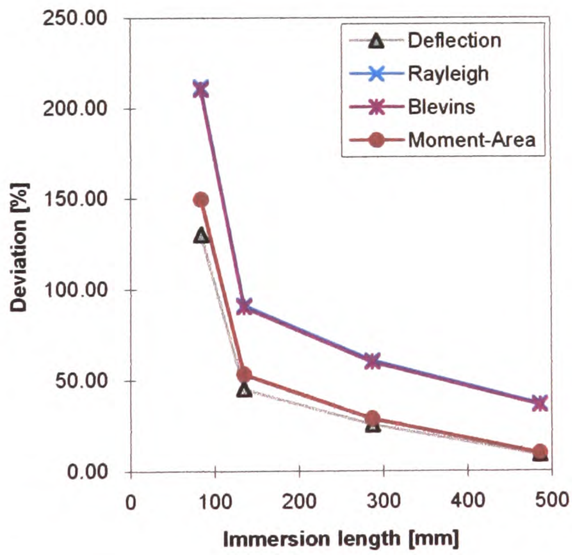


Figure 49: Deviation between experimental and theoretical results of fabricated parallel thermowells; clamped in a vice

As can be seen from the results, there is a large difference between the natural frequency established with the practical vibration analysis and the calculated values. This confirms the previous assumption that the method of clamping the thermowell influences the measured frequency. It also suggests that in order to measure the natural frequency of a thermowell accurately a test rig has to be designed that will not influence the vibration characteristic of the thermowell.

2.2.1.3.3 Test rig

A suitable rig must have the following characteristics

- thermowell has to be clamped at every point on its circumference
- vibration of rig can be neglected
- different thermowells can be fitted to the rig
- rig can be fitted to the frame available in the laboratory (Figure 50)

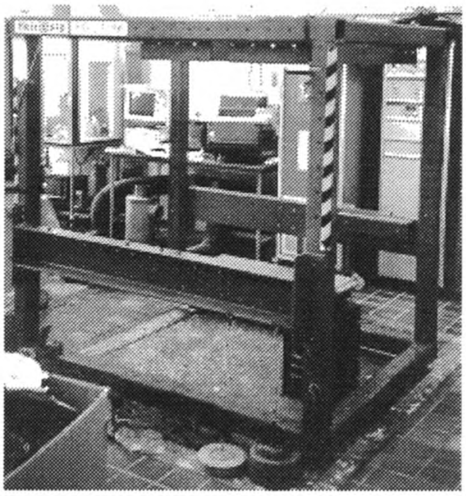
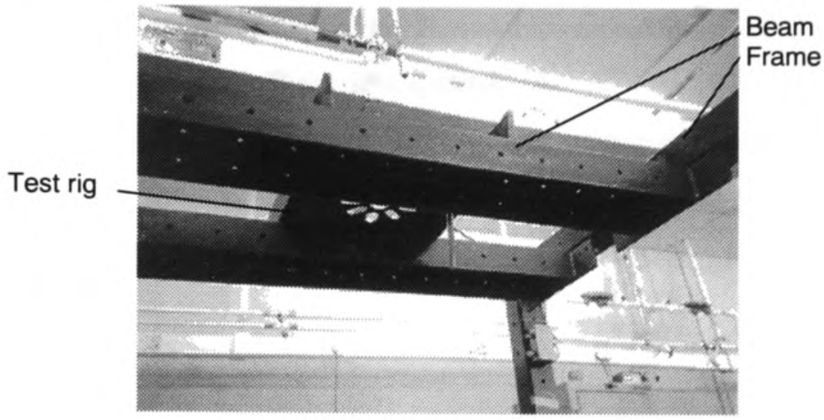
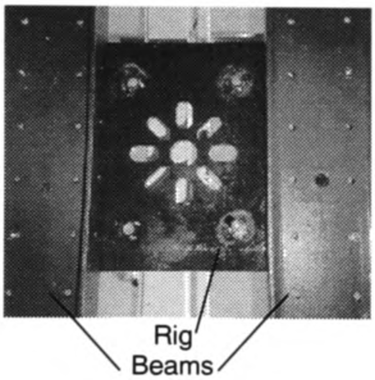


Figure 50: Frame for vibration tests

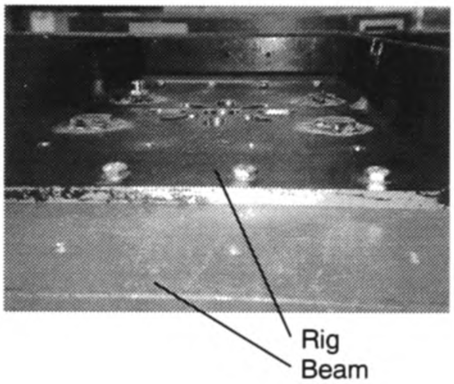
Taking these points into consideration, a rig was designed using a standard mild steel RSJ (rolled steel joist). The RSJ was modified to provide fittings for four different threads and an 'universal' flange. The threaded connections are suitable for thermowells with a ½" or ¾" BSP or NPT connection. The 'universal' flange is based on a similar design used at British Rototherm to carry out pressure tests on thermowells. Using appropriate adapters, commonly used flanges can be fitted to the rig. The rig is fixed to two beams that are attached to the top of the frame, see Figure 51 a)-c). This arrangement allows easy access for the testing of the thermowells.



a) Test rig fixed to frame



b) underside of rig



c) top of rig

Figure 51: Test rig

The combination of test rig, steel beams and frame provides a high rigidity and will therefore minimise the effect on the thermowell's natural frequency. The provision of four threaded connection and the 'universal' flange allows the mounting of thermowells rigidly to the test rig, therefore avoiding the situation encountered when using a vice.

Using this test rig, the natural frequency of the thermowells that were used in the preliminary tests was established again. Table 9 compares the results achieved when clamping the thermowells in the vice and when fixing them to the test rig.

Table 9: Comparison between tests using a vice and the rig

Nominal length [mm]	Immersion length [mm]	Vice [Hz]	Rig [Hz]	Deviation [%]
100	84	662.00	1394.00	-52.51
150	135	418.00	604.00	-30.79
300	287	110.00	154.00	-28.57
500	486	45.00	56.63	-20.54

For the calculation of the deviation, the frequencies established in the test rig are used as the reference values. There is a considerable difference in the measured natural frequencies; for the thermowell with 100mm immersion length the natural frequency when using the test rig was over twice as high as when using the vice. The results are represented in graphical form in Figure 52 and Figure 53.

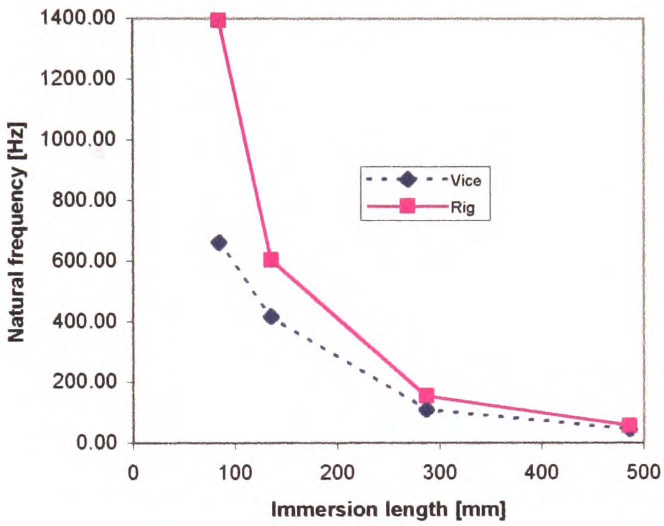


Figure 52: Natural frequency of thermowells fixed in a vice and the test rig

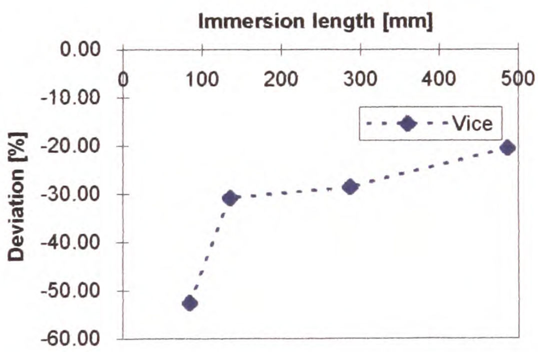


Figure 53: Deviation between results achieved in vice and in test rig; rig as reference

These results again support the assumption that the method of holding the thermowells during the tests is of importance. Furthermore, this also questions the results achieved by Murdock (1959), where it is suggested that the theoretical results are within 15% of the experimental values. However, in the paper it is only stated that thermowells "were mounted in a vise[4] and struck". There is no description of the way the thermowells were actually mounted or of the tests, therefore it is not possible to comment on the quality of the results given in the paper.

2.2.1.3.4 Test setup

The thirteen thermowells available for testing were analysed using the modal analysis software package STAR which requires a wire-frame model for each thermowell. For the tapered thermowells and the parallel thermowells with the 16mm outside diameter six points were defined on the thermowell's circumference, at three locations along the immersion length. For the parallel thermowells with the 10.5mm outside diameter only four points were used on the circumference, but again at three locations along the immersion length. See Figure 54 for typical wire-frame models of the thermowells. The models appear not to be cylindrical because STAR only uses straight lines for a graphical representation. However, it is possible to define cylindrical elements and these elements were used to the thermowell models. The points on the thermowell that are used for the excitation are numbered 1 to 18 (1 to 12 for the 10.5mm outside diameter thermowells). These 12/18 points were also marked on the individual thermowells.

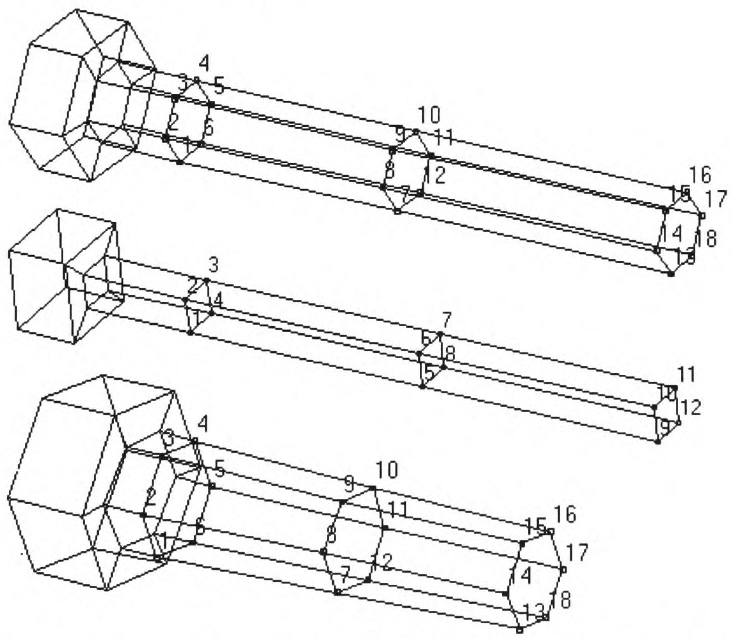


Figure 54: STAR models for parallel and tapered thermowells

[4] spelling from Murdock (1959)

Note that the STAR models are made up of more than the 12/18 points described above. The additional points are used to model the threaded connection between thermowell and test rig. They are not used for measurement or excitation as they represent the fixed end and therefore have no degree of freedom.

In order to carry out the analysis an accelerometer has to be fixed to one of the points on the thermowell. Then, each of the points is used as an excitation point and the response signal is stored in the computer's memory. Once all measurements have been taken, the STAR software package carries out the modal analysis, resulting in the values of the thermowell's natural frequency (or frequencies if more than one mode of vibration can be determined) and the appropriate mode shape(s). Figure 55 shows a schematic diagram for the set-up of the tests.

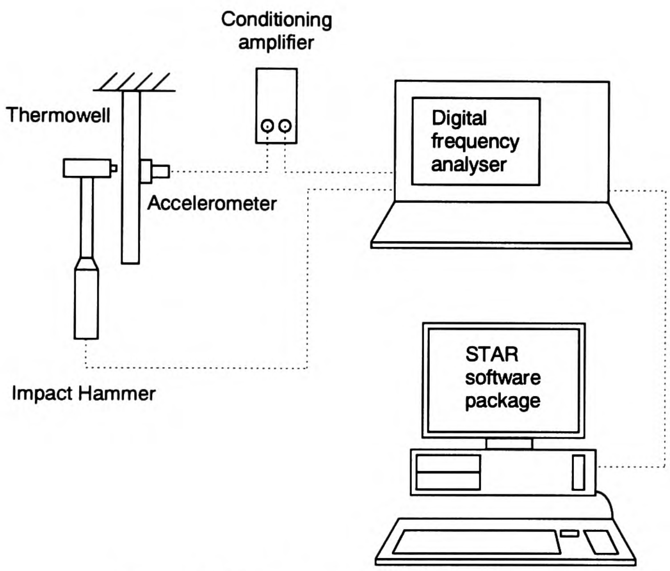


Figure 55: Schematic of setup for tests

Figure 56 shows a thermowell (in this case a parallel fabricated thermowell with a nominal length of 100mm) mounted to the test rig. The accelerometer is fixed to the thermowell with plasticine. To avoid any interference caused by the movement of the cable (i.e. *triboelectric effect*, McConnell 1997, p253) that transmits the signal from the accelerometer to the conditioning amplifier, the cable is also fixed at two points with plasticine.

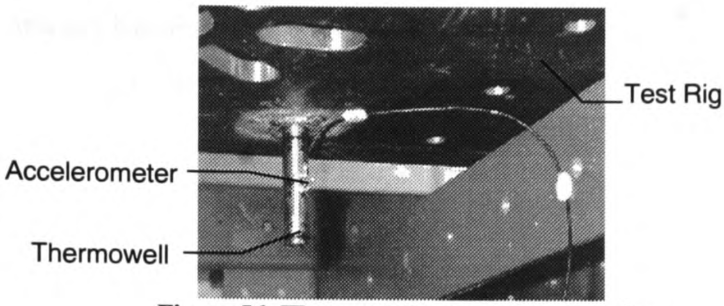


Figure 56: Thermowell mounted on test rig

The accelerometer is attached to the thermowell at the position shown in Figure 56 during all the tests. The measurements are taken about halfway along the immersion length, i.e. on the circumference described by points 7-12 (5-8 for the 10.5mm outside diameter thermowells). The exact point number varies between the individual thermowells because there was no fixed reference used to mark the points on the thermowell. At this position, the vibration of the thermowell can be measured accurately whilst the effect of the added mass of the accelerometer on the vibration can be neglected. Choosing a position close to the root of the thermowell would eradicate added mass effects, however the accuracy of the measurements would be lower. Positioning the accelerometer at the tip of the thermowell would increase the measurement accuracy but at the same time the results would be falsified by the added mass of the accelerometer.

2.2.1.3.5 Test procedure

Before testing each thermowell the measuring system was checked using the calibration exciter. The exciter vibrates at a specific frequency and when the accelerometer is fitted to the exciter it can be established whether the measuring system identifies the correct frequency. If this is the case the new thermowell can be analysed; otherwise, each component of the measuring system has to be checked and appropriate changes to the settings made to achieve the correct reading. However, this should only be necessary if different components of the measuring system are used for different tests.

The accelerometer is attached to each thermowell at the location described under *Test Setup*. The location of the accelerometer is not changed during a test.

Using the impact hammer, the thermowell has to be tapped at each of the points (Figure 57) used to define the wire-frame model in STAR.

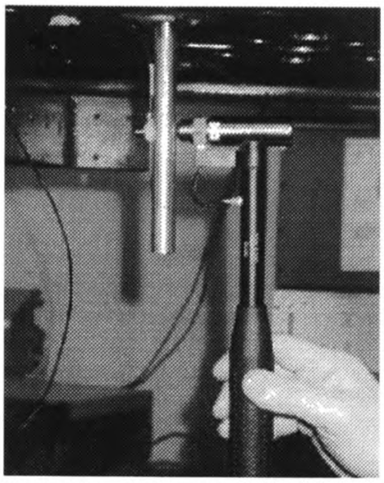


Figure 57: Excitation of thermowell with impact hammer

Each point will be excited nine times and the average of the nine measurements will be passed on to the computer. The averaging is carried out by the digital frequency analyser and is necessary in order to cancel out the effects when, for example, the tapping does not occur directly perpendicular to the thermowell surface but at a slight angle or when the thermowell is tapped above or to the side of the specified point.

When the excitation has to be performed at the location of the accelerometer, the thermowell has to be tapped at the point opposite to the accelerometer. This has to be indicated to the STAR software by adding a '-' sign before the required measurement point. STAR always prompts which excitation/measurement point combination is required and the '-' sign is a pointer for STAR that the signal is taken from the opposite direction.

Once all measurements have been taken the STAR software carries out the analysis using the provided experimental data which results in the natural frequency of the thermowell, together with the mode shapes.

This process is repeated for all available thermowells.

2.2.1.4 Results

Table 10 to Table 12 show the values for the natural frequency determined using the practical vibration analysis (column **Test**) as described in the previous section. Also shown in the tables are the values for f_n calculated with different theoretical approaches, and the deviation of these values from the practical values. Graphs of the results and the deviations are shown in Figure 58 to Figure 63.

2.2.1.4.1 Fabricated parallel thermowells, set one

As expected, the theoretical values calculated using the deflection method and the moment-area method are equal because they are based on the same equation and the only change in properties occurs at the last 3mm of the thermowell. As has been shown before, the effect of this step change can be neglected.

The values calculated using the approach proposed by Blevins (1979) and Rayleigh's method (Harris 1994) are similar, too. Both methods use the same equation, with the constant in Blevins' equation being given with more significant figures thus resulting in lower values for f_n.

Because of the similarity between the theoretical models, only Rayleigh's method and the deflection method are considered in the following argument.

Table 10: Comparison between experimental results and various theoretical approaches to establish the natural frequency of parallel thermowells; first set of thermowells

Nominal length [mm]	Immersion length [mm]	Test [Hz]	Rayleigh [Hz]	Blevins [Hz]	Moment-Area [Hz]	Deflection [Hz]
100	84	1487.00	2063.40	2055.20	1524.43	1524.43
150	135	658.88	798.90	795.70	607.36	607.36
300	287	158.32	176.80	176.10	138.05	138.05
500	486	54.09	61.60	61.40	48.64	49.64
			Deviation from test results [%]			
			38.76	38.21	2.52	2.52
			21.25	20.77	-7.82	-7.82
			11.67	11.23	-12.80	-12.80
			13.88	13.51	-10.08	-10.08

A comparison between the theoretical and the practical results shows that for immersion lengths smaller than 300mm the values calculated with the deflection method are much closer to the actual values than the values calculated using

Rayleigh's method. For immersion lengths larger than 300mm, the absolute deviation between the practical and theoretical values using either approach is similar (Table 10, Figure 58, Figure 59).

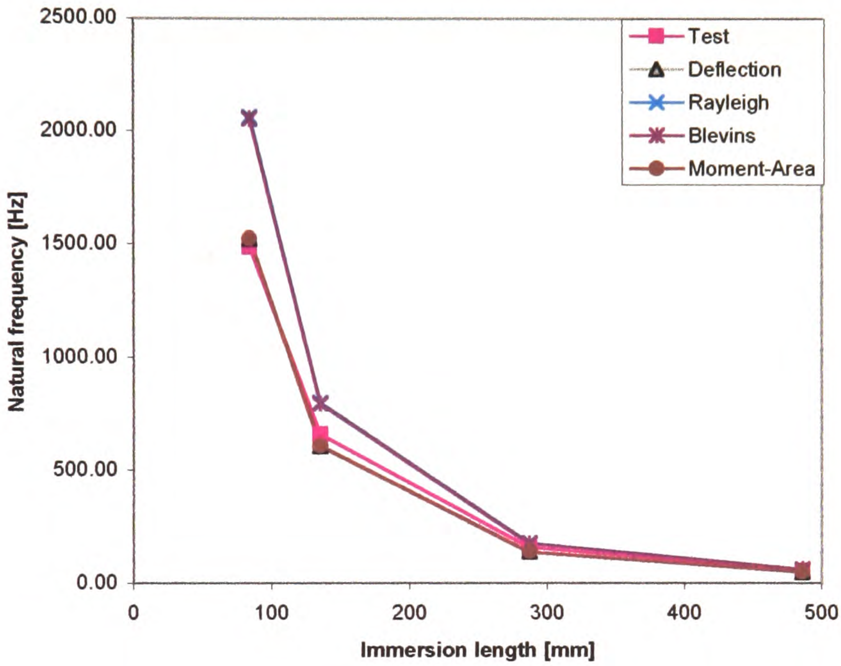


Figure 58: Natural frequency of fabricated parallel thermowells; first set of thermowells

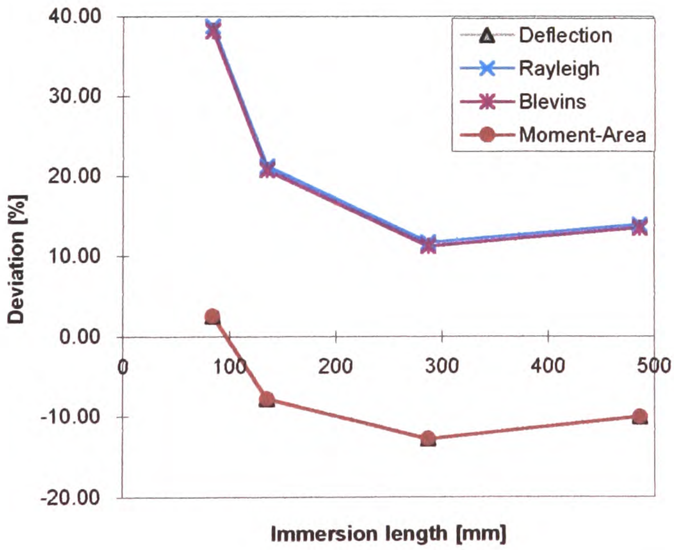


Figure 59: Deviation between experimental and theoretical results of fabricated parallel thermowells; first set of thermowells

2.2.1.4.2 Parallel fabricated thermowells, set two

As was seen for the first set of parallel thermowells, the results using Rayleigh's (Harris 1994) and Blevins' (1979) method are similar, as are the results using the moment-area and deflection methods (see Table 11). Again, only Rayleigh's method (Harris 1994) and the deflection method will be considered for the comparison with the practical results.

Table 11: Comparison between experimental results and various theoretical approaches to establish the natural frequency of parallel thermowells; second set of thermowells

Nominal length [mm]	Immersion length [mm]	Test [Hz]	Rayleigh [Hz]	Blevins [Hz]	Moment-Area [Hz]	Deflection [Hz]
100	88	964.48	1122.77	1118.32	868.58	868.58
150	139	402.54	450.01	448.23	352.45	352.45
300	287	98.60	105.56	105.14	83.63	83.63
500	488	38.68	36.51	36.37	29.10	29.10
			Deviation from test results [%]			
			16.41	15.95	-9.94	-9.94
			11.79	11.35	-12.44	-12.44
			7.06	6.63	-15.18	-15.18
			-5.61	-5.97	-24.88	-24.88

For this set of thermowells, the tendency of the deviation between theoretical f_n values and experimental values is similar to the one observed for the first set of parallel thermowells. However, the deflection method is only closer to the actual values for immersion lengths smaller than 150mm. The absolute deviation will increase for larger immersion lengths whilst it constantly decreases when using Rayleigh's method (Harris 1994), up to an immersion length of 400mm. At larger lengths, the absolute deviation between practical values and Rayleigh's method (Harris 1994) will increase again.

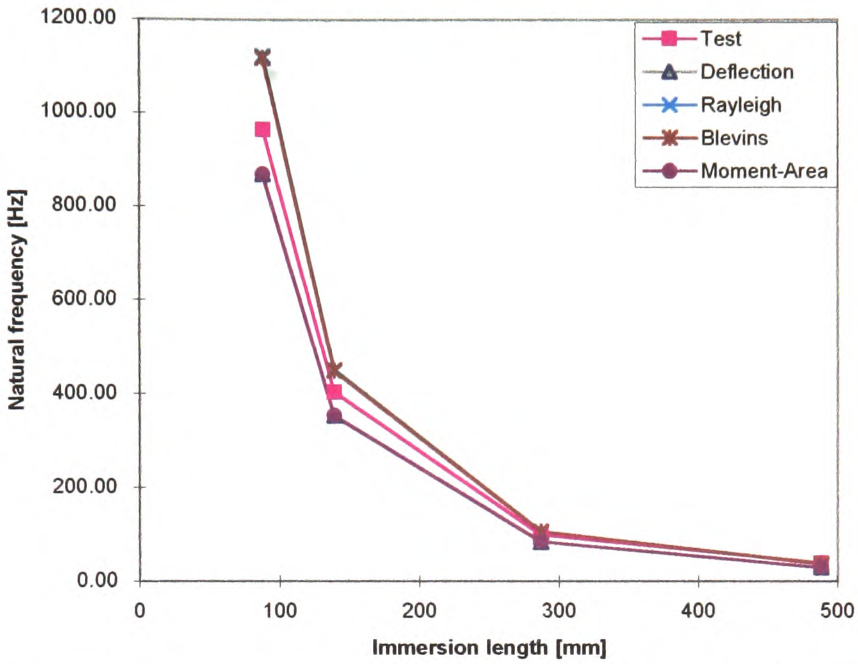


Figure 60: Natural frequency of fabricated parallel thermowells; second set of thermowells

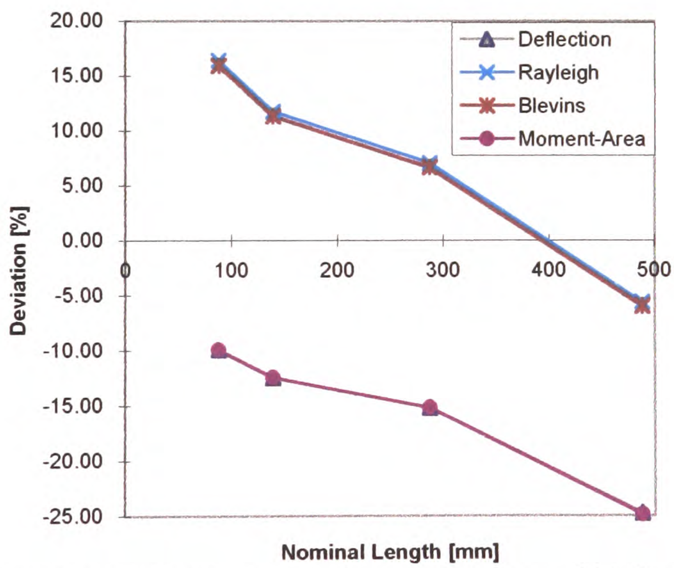


Figure 61: Deviation between experimental and theoretical results of fabricated parallel thermowells; second set of thermowells

2.2.1.4.3 Solid tapered thermowells

Table 12 shows the results for the natural frequency of tapered thermowells using the practical vibration analysis and different theoretical approaches. The geometries of the thermowells being tested are not given in the table for the constant K_f required for the method proposed by Murdock (1959) and the ASME PTC19.3 (ASME 1974). It is therefore necessary to interpolate between values given in the appropriate table in the PTC. Hence, a deviation between the practical results and the results using this method can be expected.

Table 12: Comparison between experimental results and various theoretical approaches to establish the natural frequency of solid tapered thermowells

Nominal length [mm]	Immersion length [mm]	Test [Hz]	PTC19.3 [Hz]	Integral Equation [Hz]	Blevins [Hz]	Moment-Area [Hz]	Deflection [Hz]
100	75	1928.00	2819.80	1857.70	3136.50	2303.71	2501.40
150	125	844.05	1015.10	668.80	1121.14	829.34	910.50
200	175	467.46	520.00	341.20	576.10	423.13	466.90
300	275	197.66	211.40	138.20	223.30	171.35	190.00
400	375	112.04	113.70	74.30	125.50	92.15	102.40
			Deviation from test results [%]				
			46.26	-3.65	62.68	19.49	29.74
			20.27	-20.76	32.83	-1.74	7.87
			11.24	-27.01	23.24	-9.48	-0.12
			6.95	-30.08	12.97	-13.31	-3.87
			1.48	-33.68	12.01	-17.75	-8.60

Examining the deviation between practical results and results achieved using the theoretical approaches (Table 12, Figure 63) shows a similar trend to the one observed for parallel thermowells.

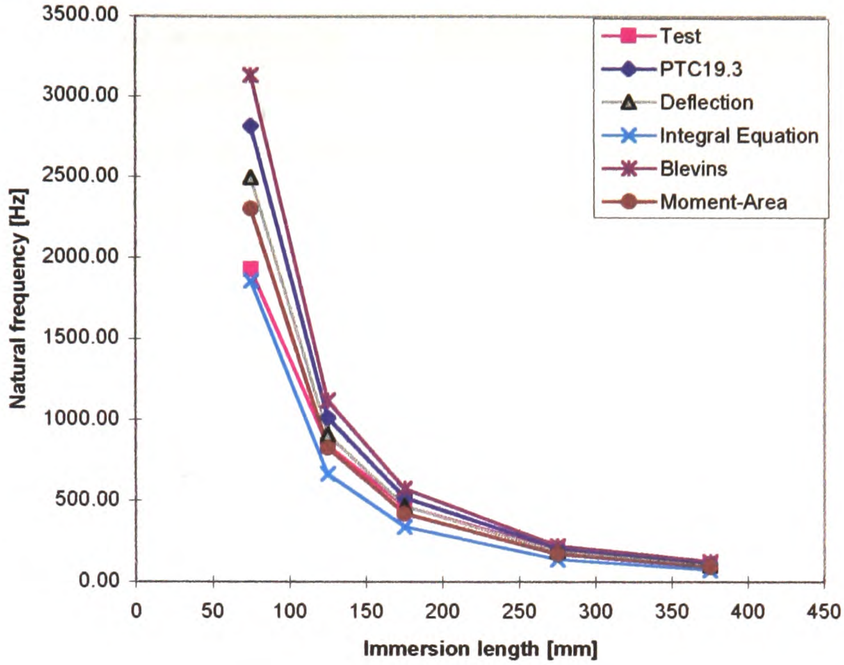


Figure 62: Natural frequency of solid tapered thermowells

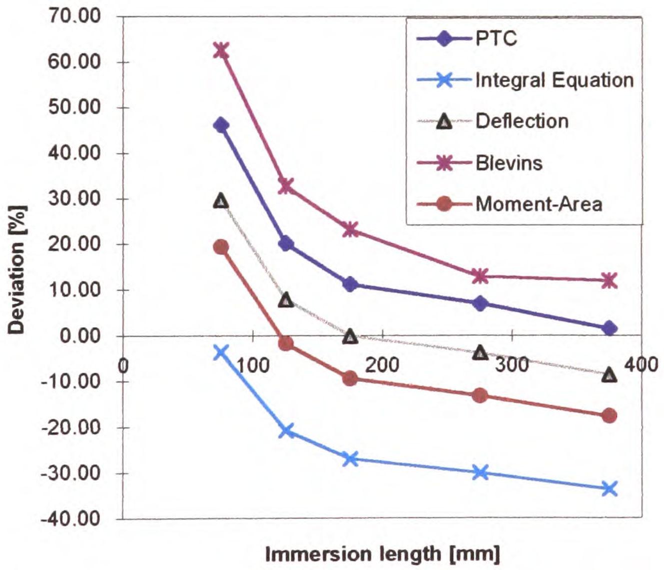


Figure 63: Deviation between experimental and theoretical results; solid tapered thermowells

Using Blevins' method (1979) results in the largest deviation between the values for the natural frequency, even though it decreases with increasing immersion length.

The behaviour is similar for the method suggested in the ASME PTC (1974), however the deviation is smaller than with Blevins' method (1979). It is only 1.5% for an immersion length of 375mm.

The deviation when using the deflection method is even smaller. However, after being almost zero for an immersion length of 175mm the absolute value of the deviation increases again.

The values calculated using the moment-area method are different to the values calculated with the deflection method, unlike for parallel thermowells. The reason for this is simply that Murdock (1959) used a concentrated load to represent the weight of the thermowell and not a uniformly distributed load. For the moment-area method a uniformly distributed load is used. Therefore, the calculated natural frequencies and the deviations are different. The values calculated using the moment-area method are closer to the actual values for immersion lengths smaller than 125mm. With increasing immersion length the absolute deviation increases again.

Finally, the results produced with the integral equation approach are smaller than the experimental results. For an immersion length of 75mm the deviation is just under 4% lower than the actual value, but the absolute deviation increases with increasing immersion length and reaches over 33% for an immersion length of 375mm.

2.2.1.4.4 Comparison with experiments carried out by Blevins *et al.* (1996)

Because of time constraints and limitations on the budget available to produce thermowells for testing it was decided to also compare values for f_n calculated with the deflection method with the experimental results produced by Blevins *et al.* (1996). The practical results are already available in Blevins *et al.* (1996), therefore it was only necessary to carry out the calculations. The thermowells used for the tests were made of either 304 or 316 grades of stainless steel and the tests were carried out at room temperature. Thermowells with the geometries given in Table 13 and Table 14 were tested. The column **Thermowell number** refers to the numbers specified by Blevins *et al.* (1996).

Table 13: Parallel thermowells tested by Blevins *et al.* (1996)

Thermowell number	Outside diameter [inch]	Bore diameter [inch]	Immersion length [inch]
1	0.752	¼	7
2	0.750	¼	10
3	0.752	¼	10
5	0.878	$^3/_8$	10
7	0.755	¼	13
8	0.760	¼	16

Table 14: Tapered thermowells tested by Blevins *et al.* (1996)

Thermowell number	Root diameter [inch]	Tip diameter [inch]	Bore diameter [inch]	Immersion length [inch]
6	0.867	0.633	¼	10
11	1	0.769	$^3/_8$	9

The results from the tests and from calculations using the deflection method are given in Table 15 and Table 16.

Table 15: Experimental and theoretical results for parallel thermowells

Thermowell number	Test [Hz]	Calculation [Hz]	Deviation [%]
1	407.07	358.5	-11.93
2	207.90	175.4	-15.63
3	205.63	175.8	-14.51
5	238.77	211.5	-11.42
7	124.04	104.3	-15.91
8	83.18	69.3	-16.65

Table 16: Experimental and theoretical results for tapered thermowells

Thermowell number	Test [Hz]	Calculation [Hz]	Deviation [%]
6	278.29	226.1	-18.75
11	377.88	327.7	-13.28

The calculated values are lower than the experimental values throughout. The absolute deviation is in the range 10%-20%.

In the case of parallel thermowells the deviation given in Table 15 confirms the findings for the first and second set of parallel thermowells where the deviation is between -10% and -22% in the same immersion length range (7" - 16", i.e. 178mm - 406mm). Figure 64 illustrates this; the immersion length range is indicated by the dashed lines.

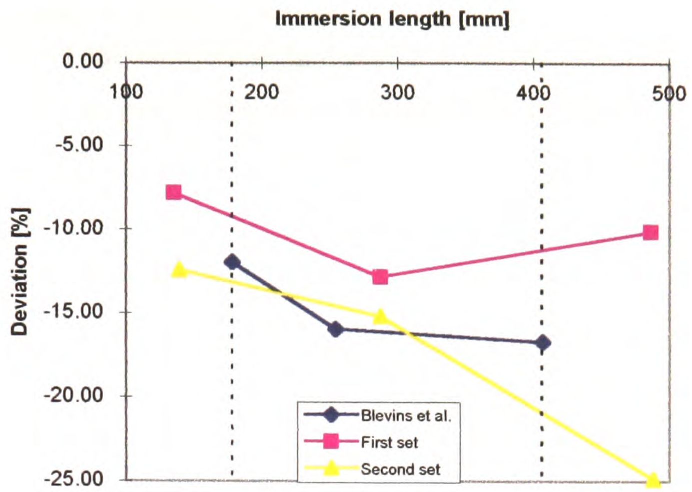


Figure 64: Comparison of the deviation for parallel thermowells

There is a larger difference in the deviation between the theoretical values for the two different sets of tests of tapered thermowells (Figure 65) in the range 6" - 7" (152.4mm - 177.8mm).

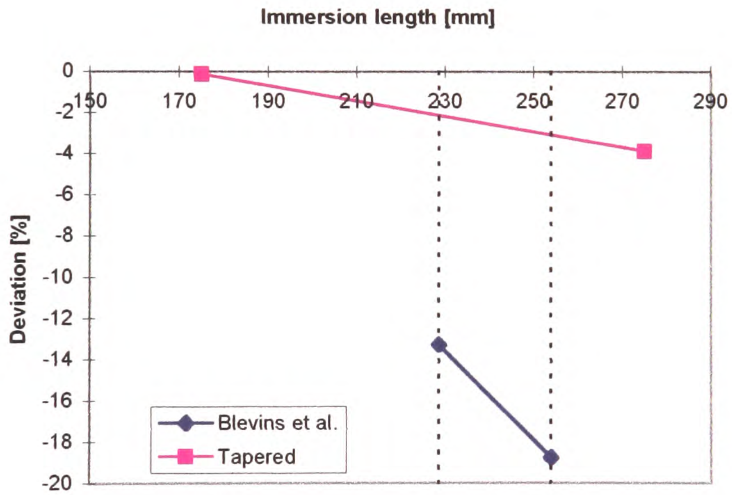


Figure 65: Comparison of the deviation for tapered thermowells

However, for both the parallel and tapered thermowells different geometries are used in the tests. Therefore, a direct comparison between the tests is not possible. However, these comparisons give a good indication for the validity of the deflection method used for the calculation of f_n.

2.2.1.5 Conclusions

From the results of the theoretical and practical vibration analysis it can be seen that it is not possible to predict the natural frequency of a thermowell with 100% accuracy. None of the theoretical approaches for the calculation of the natural frequency achieves values that are identical to the values measured, or have a constant deviation from the measured natural frequencies.

The tests using two different sets of thermowells showed that the deviation between measured and calculated values not only depends on the length of the thermowell, but also on the diameters of the individual thermowells. This observation is confirmed by calculating f_n for thermowells that were used in experiments carried out by Blevins *et al.* (1996).

Looking at the deviations between the experiments and the calculations using different methods, it can be concluded that the 'deflection' method predicts the natural frequency of thermowells with a higher accuracy than any of the other methods. With the exception of two thermowells, the deviation of the calculations from the experimental values is within ±20%. It was therefore decided to implement this method in the expert system.

2.2.2 Pressure and Stress Analysis

Methods to determine the allowable pressure and the stresses occurring in the thermowell have also been established by Murdock (1959), and as they are part of the ASME PTC19.3 (ASME 1974) the design of thermowells will be carried out according to these methods.

The equation to calculate the maximum allowable pressure a thermowell of given geometry can be exposed to is calculated using equation [4]:

$$P = K_1 \cdot S$$

$$\text{with } K_1 = \frac{B^2 - d^2}{2 \cdot B^2 \cdot F_B}$$

As was pointed out earlier, a similar equation was proposed by Roughton (1965), however with the slight difference that the factor $1/F_B$ is not used.

The Murdock (1959) equations were chosen for two reasons:

- commercial reasons; customers quote ASME PTC19.3 as a requirement
- the equations were based on pipes under external pressure. Therefore, the equation is suitable for analysis of fabricated thermowells (made of tubing), and it gives an extra margin of safety when analysing solid thermowells.

The values for F_B depend on the ratio $\phi = {}^{B-d}/_{2B}$ and are given in Murdock's paper (1959) in tabulated form (see Table 17). To simplify the determination of F_B, an equation has been established from a plot created using the values given in the table (see Figure 66).

Table 17: Table to establish F for a given ϕ

ϕ	F
0.084	2.0
0.092	1.9
0.10	1.8
0.115	1.7
0.13	1.6
0.15	1.5
0.17	1.4
0.20	1.3
0.22	1.2
0.24	1.1
0.25	1.0

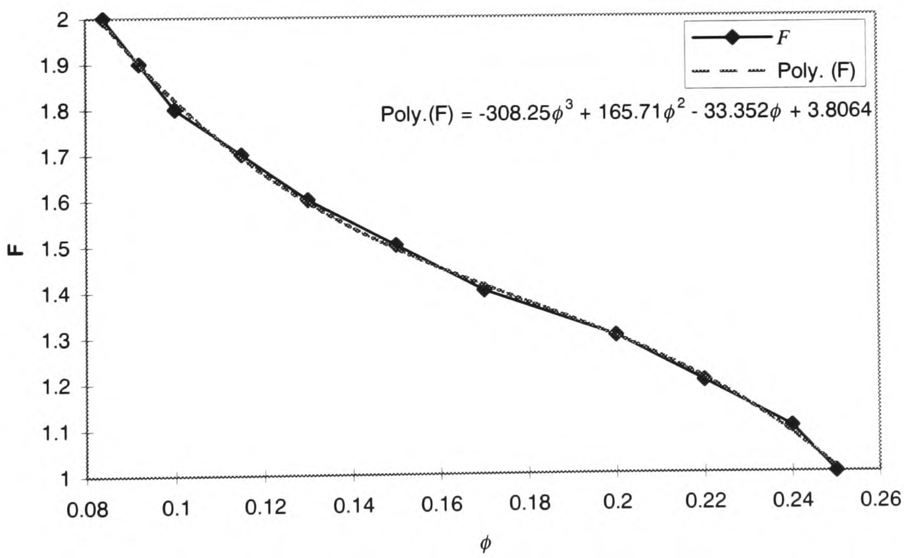


Figure 66: Graph representing values for F and the appropriate trend-line

The equation for the trend-line of the plot for F is[5]

$$F_B = -308.25\phi^3 + 165.71\phi^2 - 33.352\phi + 3.8064$$

$$\text{with } \phi = \frac{B-d}{2 \cdot B}.$$

To calculate the maximum length possible, resulting from the stresses caused in the thermowell by pressure and vibration, the equation

$$L_{\max} = \frac{K_2}{v}\sqrt{\frac{\upsilon(S - K_3 P_O)}{1 + F_M}}$$

is used. Roughton (1965) also presents a different equation for establishing the maximum allowable length. But as was discussed early, the Roughton (1965) equation should only be used for thermowells immersed in fluids of low density. The above equation can be used for any kind of fluid (see section 2.1.2).

The value of the constant K_2 as calculated with the equation given by Murdock (1959) has the dimension of feet. It can therefore be argued that when directly calculating K_2 with the given equation

$$K_2 = \sqrt{\frac{9262}{F_A \cdot K_x}}$$

the set of units used will affect the value of K_2. A comparison has been carried out using the specified units and metric units to calculate K_2, K_3 and $L_{\max}$; the result can be seen in Table 18. The thermowell details were taken directly from the example used by Murdock (1959):

Thermowell size IV, A = 1", B = $^{13}/_{16}$", d = $^{7}/_{16}$", L = 4.5", S = 13100psi = $90.3 \cdot 10^7$Nm^{-2}, P_O = 2400psi = 165bar, v = 300fts^{-1} = 91.44ms^{-1}, r = 0.656, υ = 0.335ft^3/lb = 0.021m^3/kg.

Table 18: Comparison between metric and imperial units using the same equation

	Imperial units	Metric units
F_A	1	1
K_a	1.237	1.237
K_x	4.672in^{-2}	7171m^{-2}
K_2	44.7"	1.136m
K_3	0.237	0.237
$L_{\max}$	7.3"=185mm	12.62m

[5] Established using the trend-line feature of EXCEL

It can be seen from Table 18 that all dimensionless constants are equal and that the constants K_x and K_2 which are affected by dimensions can be directly converted from imperial units to metric units and vice versa, i.e. $4.672\text{in}^{-2} \equiv 7171\text{m}^{-2}$ and 44.7" = 1.136m. However, when looking at the calculated maximum length L_{max} it is obvious that it is not possible to use metric units with the equation in its current form because the calculated lengths are completely different. Consequently it has to be investigated how the equation can be written in order to avoid incompatibilities with a different set of units.

The stress correction factor F_A is dimensionless and therefore will not be of influence when considering different units. K_x has units of in^{-2}, as it depends on the outside and bore diameters which have to be specified in inches. Because it is required to specify the diameters in inches, no conversion has to be carried out and K_x therefore has no influence on the calculation of K_2 in a different set of units, either.

The constant K_2 is derived partially from the velocity pressure (Murdock 1959), the equation for which is

$$P_v = \frac{v^2}{9262\upsilon}$$

Similar to the equation for the wake frequency as proposed by Murdock (1959), this equation also contains a factor which will ensure that the result has the correct units (in this case psi) when using the indicated units for v (ft/s) and υ (ft^3/lb).

The velocity pressure has been calculated by dividing the impingement force by the projected well area (Murdock 1959):

$$P_v = {}^F/_A$$

$$F = \tfrac{1}{2} A C_d \rho v^2$$

$$\rho = {}^1/_\upsilon \text{ and } C_d = 1 \text{ (Murdock 1959)}$$

$$P_v = \frac{v^2}{2\upsilon}$$

This equation is identical to the equation for the dynamic pressure given in the fluid dynamics literature, for example Bohl (1991, p84):

$$P_{dyn} = \frac{\rho v^2}{2} = \frac{v^2}{2\upsilon}$$

Therefore, the equation proposed by Murdock contains a conversion factor of 4631 with unspecified units. Carrying out a unit check using SI-units it can be seen that the equation for P_v indeed represents a pressure, as it has units of $\mathrm{Nm^{-2}}$:

$$P_v = \frac{v^2}{2\upsilon} \quad \left[\frac{\mathrm{m^2/s^2}}{\mathrm{m^3/kg}} = \frac{\mathrm{kg}}{\mathrm{m \cdot s^2}} \middle| \cdot \frac{\mathrm{m^2}}{\mathrm{m^2}} = \frac{\mathrm{N}}{\mathrm{m^2}} \right]$$

A similar unit check using the equation and units suggested by Murdock (1959) is carried out:

$$P_v = \frac{v^2}{9262\upsilon} \quad \left[\frac{\mathrm{ft^2/s^2}}{\mathrm{ft^3/lb}} = \frac{\mathrm{lb}}{\mathrm{ft \cdot s^2}} \quad \left(= \frac{\mathrm{lb}}{\mathrm{in \cdot s^2}} \right) \right]$$

This clearly is not the expected unit of psi. However, it is necessary to convert lb, which represents a mass, into $\mathrm{lb_f}$, i.e. pounds-force, first. The relationship between pounds-force and pounds-mass is $\mathrm{lb_f} = \mathrm{lb} \cdot g$. In the imperial system of units, the acceleration of gravity is $g = 32.16\mathrm{ft/s^2}$. Therefore, the units check changes as follows

$$P_v = \frac{v^2}{9262\upsilon} \quad \left[\frac{\mathrm{ft^2/s^2}}{\mathrm{ft^3/lb}} = \frac{\mathrm{ft^2/s^2}}{\mathrm{ft^3} \Big/ \frac{\mathrm{lb_f}}{\mathrm{ft/s^2}}} = \frac{\mathrm{ft^2 \cdot lb_f \cdot s^2}}{\mathrm{s^2 \cdot ft^3 \cdot ft}} = \frac{\mathrm{lb_f}}{\mathrm{ft^2}} \quad \left(= \frac{\mathrm{lb_f}}{\mathrm{in^2}} \right) \right]$$

Therefore, when using imperial units the equation

$$P_v = \frac{v^2}{2\upsilon}$$

has to be divided by the acceleration of gravity $g = 32.16\mathrm{ft/s^2}$ and the conversion factor to convert $\mathrm{ft^2}$ to $\mathrm{in^2}$, $144\mathrm{in^2ft^{-2}}$, resulting in a factor of $4631\mathrm{in^2/(s^2ft)}$. This factor has the same value as suggested above. Taking the ½ into account as given in the equation for P_v, the resulting factor is 9262. This is the same factor as given by Murdock (1959). This suggests that the equation for K_2 for metric units can be written in the form:

$$K_2 = \sqrt{\frac{2}{F_A \cdot K_x}}$$

The other necessary constants can be used with metric units in the form given by Murdock (1959):

$$K_3 = F_A(K_a - 1)\,,\ K_x = \frac{1.698(A+2B)A}{A^4 - d^4}\,,\ K_a = \frac{A^2}{A^2 - d^2}$$

The value of the stress correction factor F_A that is required to calculate the constants K_2, K_3 is established using the equation for F_B as given above, but using $\phi = {}^{A-d}/_{2A}$ instead as it refers to the conditions at the root of the thermowell.

A comparison was again carried out to validate the above equations, using the same example as in the previous comparison. Table 19 details the results; it can be seen that the value for K_2 cannot be converted from one set of units to the other. However, the calculated maximum length is now correct. Therefore, the equation for K_2 can be used in the proposed form.

Table 19: Comparison between metric and imperial units using different equations

	Imperial units	Metric units
F_A	1	1
K_a	1.237	1.237
K_x	4.672in^{-2}	7171.5m^{-2}
K_2	44.7"	0.017m
K_3	0.237	0.237
L_{max}	7.3"=185mm	185mm

The discussed equations for P and L_{max}, together with the equations for the necessary constants, can be used for all three types of thermowells.

2.2.3 Geometry and Manufacture

As discussed in section 2.1.3, there are three types of thermowells (parallel, tapered and reduced parallel) and two ways of manufacture (fabricated and drilled from solid bar). Because parallel thermowells are suitable for all types of temperature sensors and fabrication is the cost effective way of manufacturing a thermowell, it is proposed to select this type and manufacturing method initially when beginning the design process. Exceptions to this rule are client specifications that a solid thermowell is used and that the process pressure is higher than the 70 bar permissible for fabricated thermowells, which requires a solid thermowell. However, if a fabricated thermowell is used, the stress analysis might show that either a different tube or a solid thermowell has to be used instead.

When a solid design is required then the initial shape will be parallel as it is the simplest shape to manufacture. Again, the vibration or stress criteria might require a change of outside diameters. In this case the type of sensor used in the application is considered, provided it has been specified. If a thermocouple is used then a tapered design is attempted; if a RTD or other stem-sensitive instruments are used then a reduced parallel design should be implemented. If no sensor has been specified then the tapered shape will be used.

The inside diameter of the thermowell also has to be considered in case a sensor has been specified. For RTDs it is suggested to have the sensitive portion of the stem as close to the thermowell walls as possible. Therefore, the recommendation by Frank (1994) is used, setting the bore diameter to be 0.25mm larger than the sensors stem diameter. If any other is specified then the bore diameter is set to be 3mm larger then the stem to allow for easy assembly.

2.2.4 Material Considerations

The implementation of procedures and rules that will enable the expert system to make a decision upon which material is most suitable for the given fluid, at given pressures and temperatures proved difficult due to the complexity of the topic, the lack of a computerised materials database and the time restrictions for this project. Also, according to UK Steel (1998) who provide their steel catalogue as a book and on CD-ROM, current materials databases do not provide information on the modulus of elasticity and the specific weight at different temperatures. They are usually only given at room temperature. Therefore, even if a computerised database was available, it is not necessarily possible to have access to all the relevant properties at the required conditions. This is in line with Kokotos, Tabeshfar and Wilson's (1993) statement that "obtaining non conflicting data from the range of sources, even for basic property values of common materials often proved difficult".

Furthermore, the increasing demand for coated thermowells makes the selection of a suitable material or coating even more complicated because there can be a large number of suitable materials. It is therefore more appropriate for the client to make a choice of material. Hence, provision is made in the expert system to allow the user to specify the material. To avoid any compatibility problems between thermowell material and process fluid, it is suggested to use the same material for the thermowell as is used for the process pipe or vessel. The material of the vessel should be known by the client. If no material has been specified then 316 stainless steel will be used, as this is the most commonly used material and offers a wide range of applications due to its high strength and good corrosive resistance even at elevated temperatures (Frank 1994; CM 1996).

2.2.5 Selection of Process Connection

It is expected that the process connection would usually be specified by the client, because it has to fit into an existing pipe or vessel. This pipe or vessel already has a provision for a thermowell and therefore the thermowell's connection has to be compatible with the existing connection.

However, if the thermowell is required for a new plant or a plant under construction it is possible that no decision on the type or size of the connection has been made yet. In this case the expert system can make a recommendation as to what type of connection should be used.

The recommendations made by Frank (1994) take several conditions, such as the requirement to remove the thermowell at regular intervals or the size of the pipe, into account. If all these conditions were to be used by the expert system to make a decision about the process connection then the user would have to provide a great deal of additional information. Also, these requirements will only surface during the operation of the process and will therefore not be known for a new plant. It was therefore decided to select the connection type by considering the process pressure only, a condition which is relevant for all three types (see section 2.1.5).

For what is considered low pressure applications, i.e. pressures less than 70 bar (Rototherm 1996), a threaded process connection will be used. For pressures higher than 70 bar and lower than the maximum flange rating of the highest rated flange available, a flange with the appropriate rating is used. The highest flange rating is 2500lb (RGB 1994), i.e. 172 bar. Should the pressure of the application exceed this rating then a weld-in socket is used.

Consideration must also be given to the size of the connection, i.e. the thread size, flange size or the diameter of the weld-in socket. These dimensions depend on the root diameter of the thermowell. When using a threaded connection, the gauge diameter of the thread must be larger than the root diameter, otherwise it is not possible to fit the thermowell. For flanges, the raised face diameter determines the suitability. Because the thermowell is welded to the flange, an allowance for the weldment must be made. Therefore, a flange with a raised face diameter 20mm larger than the root diameter should be chosen. In the case of weld-in sockets, the maximum diameter of the socket is limited by the size of the bar that is used to manufacture the thermowell. If the diameter of the bar is more than 6mm larger than the thermowell's root diameter then the socket diameter will be set to root diameter + 6mm. Otherwise, the bar's diameter is used as the socket diameter. If a hexagonal bar is used, then the diameter is the diameter across flats, therefore minimising the machining time.

When selecting the process connection, the type of the connection will be determined first, followed by the connection's size. However, if it is not possible to find a suitable size of connection, for example when the root diameter is larger than the gauge diameter of the largest available threaded connection, a different type of connection will be used. Hence, if no suitable threaded connection can be found then a flanged connection will be used. Should this also prove unsatisfactory then a weld-in socket will be used.

2.2.6 Thermal Considerations

Roughton (1965) proposes the most comprehensive method, but due to use of various graphs and no clear indication how these graphs were determined it proves difficult to implement it as an automated approach. Furthermore, the temperature and velocity profile of the process have to be known in order to establish the required local values for the heat transfer coefficient h and the mean velocity and temperature. Values for the friction factor of the pipe, which is necessary to calculate the gradient error, could not be found anywhere. This information would have to be provided by the client, together with the correct information for the radiation characteristics which could not be established sufficiently. The time constant for the temperature sensor has to be provided in order to establish the intrinsic dynamic error. Also, the requirement that the mean temperature is the temperature of interest has only been stated by Roughton (1965) and it can be assumed that it is only true for the particular application Roughton (1965) is referring to.

The iterative approach of Benedict and Murdock's (1963) stepwise linearisation of their established equations is suitable for implementation in a computer application. However, this approach also requires a considerable amount of additional information, for instance the emissivity of the fluid at different temperatures; as pointed out by Benedict and Murdock (1963) and Roughton (1965), the information on the emissivity of fluids is rare, though.

The approach suggested by Richmond (1980) allows the calculation of the measurement error caused by conduction and the response time of the thermowell. The only additional information that has to be provided is the length of the thermowell that is completely immersed in the flowing fluid. However, when trying to establish the overall heat coefficient U according to Gibson (1995) for the equation of T_e, it was not possible to achieve the same values as are indicated by Richmond (1980). It would therefore be necessary for the client to provide this information, as it currently cannot be calculated.

It was decided to implement the last approach, the method according to Richmond (1980), in the expert system. It is the only approach that requires a minimum of additional information, which is important when dealing with a client. It can be used for all liquids without restriction. If it is used for gases and vapours at temperatures higher than 120°C then it will result in a lower limit for the measurement uncertainties; a higher deviation from the actual fluid temperature can be expected. This method therefore gives an indication of the magnitude of the measurement error for all fluids. Considering that the other two approaches should not be used to make corrections to a sensor indication (Benedict and Murdock 1963), but merely indicate the range of the measurement error, this method can be used to provide the client with an indication to the thermowell's measurement performance.

The equations proposed by Richmond (1980) to calculate the temperature error and the thermowell's time constant can be used with any consistent set of units.

$$T_e = \frac{T_f - T_a}{\cosh(ML_1)}$$

$$\tau = \frac{mC_p}{UA}$$

The equation for the calculation of the heat transfer factor M has to be slightly modified, however. The proposed equation takes care of the conversion feet to inches. This is necessary because M must have units of inches in order to give a dimensionless number when being multiplied with length L_1 in the equation for T_e. Using a consistent set of metric units throughout the calculations, the equation must be used in the form:

$$M = \sqrt{\frac{\pi DU}{ka}}$$

2.3 Systematic Design Approach

2.3.1 Design Terminology

Pahl and Beitz (1988) distinguish between three types of design whose boundaries are not very distinct:

- Original design involves the development of an original solution principle for a system with the same, similar or new task.
- In adaptive design a known system in which the solution principle remains unchanged is adapted to a changed task. This requires the original design of individual parts or assemblies, however.
- Variant design involves varying the size and/or arrangement of certain aspects of the chosen system, the function and solution principle remaining the same. No new problems arise as a result of, for example, changes in materials, constraints or technological factors. This also covers what is sometimes referred to as *fixed principle design*, namely commissioned work in which the solution principle and finished design remain the same and only dimensions of individual parts are changed.

The case of thermowell design is a combination of adaptive and variant design. The process conditions change from application to application (or even within an application if thermowells for more than one vessel are required), but the solution principle remains the same. This is the case of adaptive design. Changes are only made to the dimensions and material, which suggests variant design or fixed principle design. However, those changes can create new problems, for example failure of the frequency criterion, which again suggests adaptive design. This emphasises the point that there is no clear boundary between the types of design as discussed above.

Even though a differentiation is made between types of design, the *design process* is the same in every case.

2.3.2 Design Process

The design process consists of four phases (Pahl and Beitz 1988):

- Clarification of the task

  This phase is concerned with the collection of information about the requirements and restrictions for the design. The compilation of the appropriate information is referred to as the *design brief* or *design specifications*.

- Conceptual design

  This refers to the establishment of function structures, solution principles and their combination into concept variants. Once these variants have been evaluated according to mainly technical criteria the best solution can be selected.

- Embodiment design

  In this phase the layout and form of the design have to be determined and the technical product or system has to be developed. This may result in several variants which have to be evaluated using both technical and economic criteria. The variants can be combined to create an optimum solution.

- Detail design

  In the final step all properties of all the components necessary for the product or system are specified. Also, the technical and economic feasibility has to be re-checked and all drawings and other production documents are produced.

As the different types of design are indistinct, equally the design stages are not clearly separated from each other. Pugh (1991), for example, does not include embodiment design as a separate phase in his proposed design steps, instead both the conceptual and detail design phases contain parts of the embodiment design suggested above.

Depending on the scale of the product or system to be designed the designer will be in contact with the client after or even during each design step. This is especially important during and after the conceptual design phase, because it allows the client to make changes to the specifications at an early stage when modifications to the design are carried out easily and at a low cost. When reviewing the conceptual design proposals the client might realise that some conditions have not been specified and could cause problems with the current design proposals. These problems can then be rectified by adopting the designs to the new conditions. Only after the final design has been approved by the client will the product or system be manufactured. Figure 67 shows the discussed design process which is applicable for design tasks in general.

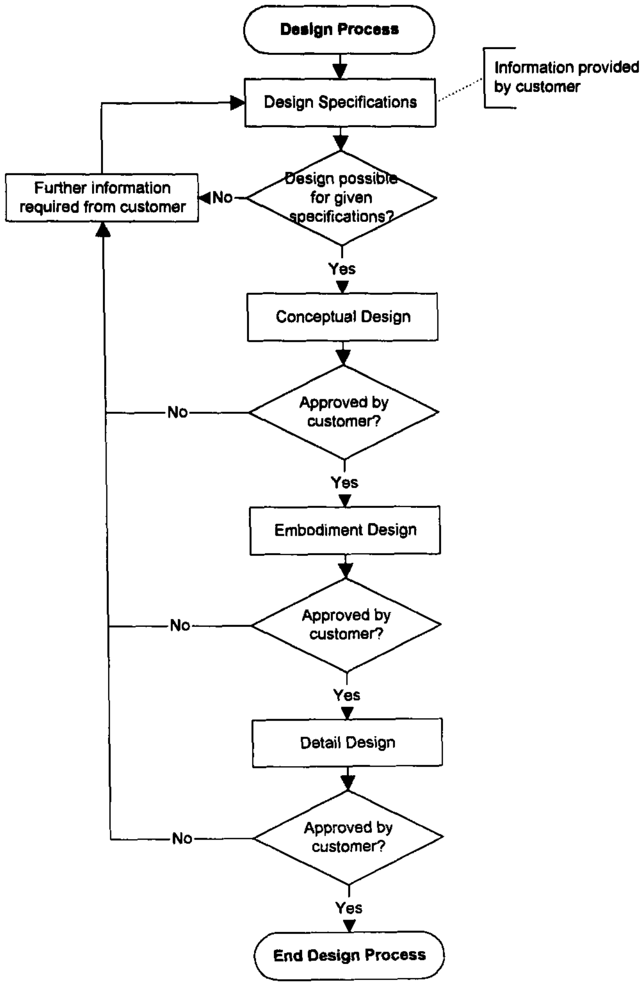


Figure 67: General design process

2.3.3 Application of the Design Process to Thermowell Design

The type of design used for thermowells has already been established, it is a combination of both adaptive and variant design. The design phases as outlined in the previous section are also applied to thermowell design:

- The design specifications will be provided by the client when placing the order. The information specified comprises the process conditions (i.e. the fluid, flow velocity, operating temperature and pressure), details about the sensor, any restrictions on the possible thermowell length or diameter and the process connection necessary to fit the thermowell to the vessel or pipe. To establish the optimum design for a given application, all of the above information has to be specified. If the client is not certain about one or more aspects of the specifications, assumptions need to be made either by the client or the thermowell designer in order to establish a design.

Depending if the assumptions are correct, somewhat correct or not correct, the proposed design will be satisfactory for the application, sufficient for a limited operation time only or not appropriate at all. For example, if the client is not aware of the operating temperature, the designer assumes a temperature that can be too high, too low or correct. If the assumption was too high or correct, the thermowell will be suitable for the application; however, a different and therefore possibly cheaper material could have been used if the operating temperature is lower than the assumed temperature. If the assumption was much lower than the actual operating temperature, the thermowell has a much shorter operating life than expected; it could even be not suitable at all in case the operating temperature is at the material's melting temperature.

Alternatively, the designer can provide the client with any restrictions that are applicable for a certain design. For instance, if the flow velocity is not known the designer can suggest a design together with the maximum allowable velocity for that particular design. The plant or process operators can then adjust the flow accordingly, provided this is possible. It can be seen from those two examples that detailed design specifications are vital to a successful thermowell design.

- The basic shapes of thermowells are well established, therefore it is not necessary to develop original concepts in the conceptual design phase. However, there are various possibilities concerning the thermowell diameters (bore, tip and root diameter), immersion length, material and the type and size of the process connection. These characteristics have to be chosen according to the design specifications. The established methods for vibration, stress and pressure analysis together with information about the material compatibility, and the temperature and pressure ranges will be used at this stage. For example, for a set of given specifications it could be possible to achieve satisfactory designs with a parallel thermowell of length L_1 and outside diameter A_1 and a parallel thermowell with $L_2 > L_1$ and $A_2 > A_1$, but with equal bore diameter $d_1 = d_2$.

- An embodiment design phase in its own right is not required for thermowell design. It is partly included in the conceptual and detail design. The information required for the evaluation of the designs can directly be taken from the concepts as they have to be detailed in order to apply the analysis methods described earlier. For example, if one concept is a fabricated thermowell then the diameters are automatically given by the tube which is used to manufacture the thermowell. Exact specification of the tube used is already required at this stage to carry out the technical evaluation concerning the stress, pressure and vibration analysis.
- In the detail design phase the design best suited for the application is selected and the appropriate drawing is produced. The drawing is usually a standard form where only the appropriate information about the dimensions and the material are added. Additional paperwork such as the results from the vibration and stress analysis or material certificates which are requested by the client in most cases, is also produced at this stage.

The contact between the designer and client will be rather limited. After the designer has received the necessary information concerning the design specifications both the conceptual designs and the detail design will be established. This can be justified because even conceptual thermowell designs have to be fully detailed (see above) and the only additional work in the detail design phase is producing the accompanying paperwork. The client will only be contacted in case that a design within the design specifications is not possible; for example, the conceptual designs could show that the thermowell diameter has to be larger than possible with the specified process connection.

If a client commissions the design and manufacture of a thermowell to a specialist instrumentation manufacturer such as British Rototherm, then the client is usually not in a position to either approve or reject a thermowell design. However, the client will still be informed about the proposed design before its manufacture in case new conditions have arisen since the design specification was first established.

Figure 68 shows the design process for thermowells as discussed above.

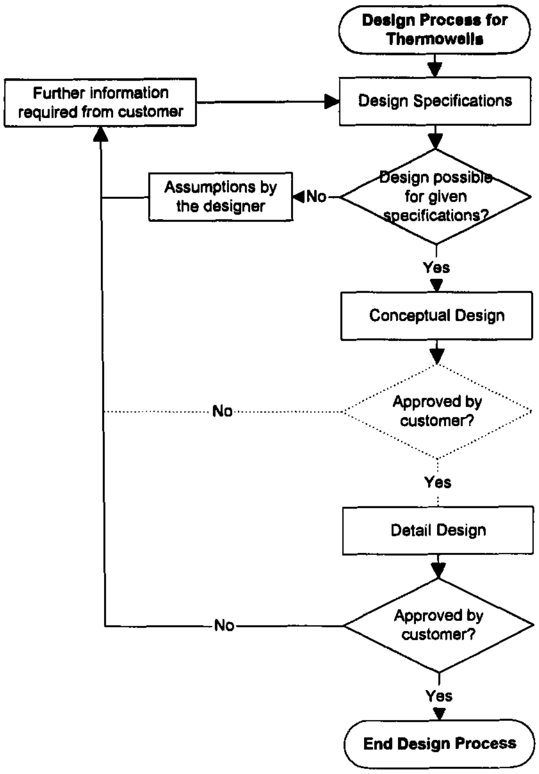


Figure 68: Design process for thermowells

3. Expert System for the Design of Thermowells

3.1 Expert Systems

Expert Systems can be described as sophisticated computer programs which are based on knowledge of a certain area, the *knowledge domain.* The knowledge used by the system to find solutions to a specific problem consists of facts and heuristics, i.e. rules of thumb, and can be described symbolically or mathematically.

The following participants are involved in the building and utilisation of an expert system (see also Figure 69):

- The knowledge of the domain expert is used by the expert system to generate solutions for the problems. The knowledge of the expert system can also include information based on books or other sources. The domain expert also tests the expert system.
- The knowledge engineer interviews the expert, encodes the expert's knowledge and implements it in the expert system. Usually, the knowledge is a set of rules in the form of IF...THEN... statements and is stored in the *knowledge base.*
- The expert system building tool is the programming language or development environment used to create the expert system.
- The user uses the expert system after its development. It can be an expert, or a novice user. The expert system is usually used in an advisory capacity.

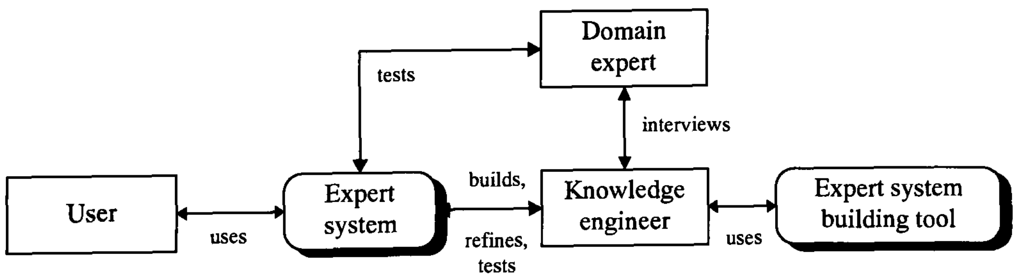


Figure 69: Relationship between participants in expert system development and application

3.1.1 Organising Expert Systems

The knowledge needed by an expert system is represented in the form of facts or rules. As it is possible that some rules or facts are not always true or false, a certainty factor can be introduced for those cases. This is usually required for expert systems performing diagnostic tasks such as diagnosing bacterial infections in hospital patients (Waterman 1986, p34). A different example which can be experienced daily is the weather forecast. The experts predicting the weather will take certain factors into consideration and from that the likelihood of rain, for instance, can be determined. This likelihood is the equivalent of certainty factors in expert systems.

The fact that rules in expert systems are usually heuristic, i.e. simplifications or rules of thumb, means that solutions will be found most of the time, contrary to an algorithmic method which guarantees a correct solution every time. The algorithmic method, however, makes the search for solutions rather complicated and the solution can be less practical, though technically correct.

The knowledge of an expert system can be divided into two categories. The general problem-solving knowledge, called the *inference engine*, is responsible for the operation of the expert system, e.g. interacting with the user or determining which rules to apply, whilst the knowledge base represents the information necessary to solve the problems the expert system was designed for.

There are several methods to represent knowledge in an expert system, but just three of them are in general use. The *rule-based* method uses the IF... THEN... statements used in most computer languages. In the case of a match between the current problem and the IF part of the statement the rule is *fired*, i.e. the THEN part will be executed. The execution of the THEN part may result in the modification of the facts in the knowledge base, for example by adding a new fact. A series of matching IF... THEN... conditions is called an *inference chain*. There are two ways of implementing such an inference chain. In *forward chaining* rules are matched against given facts in order to establish new facts. The opposite approach is called *backward chaining* where the expert system attempts to establish the facts necessary to prove a *goal* which represents the fact or conditions that have to be proven. This method is sometimes also referred to as being *goal driven*.

The second and third methods are closely related, they are both *frame based systems*. In *semantic nets* or *semantic networks* nodes, which represent concepts, objects or events, are connected by arcs which describe the relationship between the nodes. Frequently used arcs for hierarchies are ISA and HAS-PART; see Figure 70 for an example (Waterman 1986, p71).

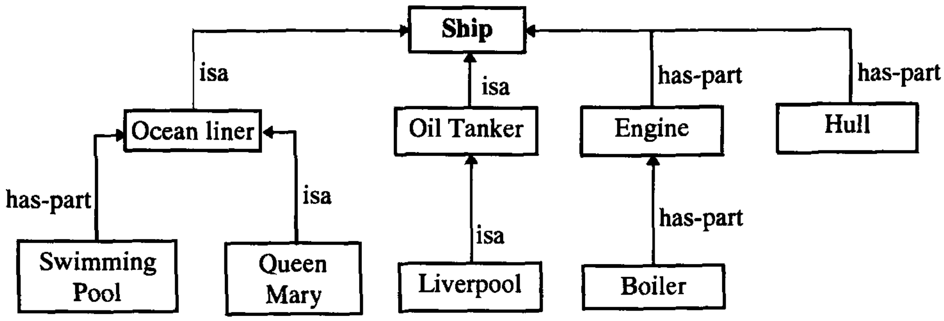


Figure 70: Example of a semantic net for the concept of a ship

For the description of natural languages arcs such as AGENT, OBJECT and RECIPIENT are used. In the hierarchy structure of semantic nets it is possible for items lower in the net to inherit properties from items higher up as these statements can be inferred from the net. Therefore, it is possible to store information about similar nodes in a central location and avoid the repetition of facts at those nodes, consequently saving space.

The other method of knowledge representation utilises *frames*. The difference between *semantic nets* and *frame systems* is in the organisation of the nodes. In frames, each node resembles a number of attributes such as names or dimensions, and values of those attributes. The attributes are referred to as slots and each slot can have procedures attached to it that are executed once the information contained in the slot is changed. Figure 71 shows an example for a frame system using the three different thermowell types.

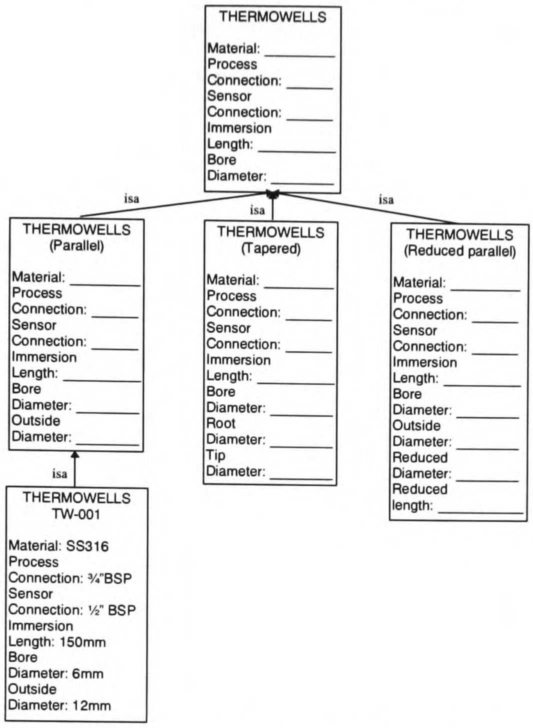


Figure 71: Example for a frame system

3.1.2 Expert Systems for Design

Expert Systems for design can be used in two ways, *design analysis* and *design synthesis*.

In design synthesis the expert system is capable of generating a design. Used for design diagnosis the expert system can evaluate a design, criticise it and recommend corrections. In other words, in design analysis a given description of an object leads to a description of its performance, whilst in design synthesis a required performance or specification leads to the description of an object (Rosenman *et al.* 1987).

Another way of using expert systems in design related applications is by supporting the designer in the decision-making process of a certain design aspect. For example, the Total Design Manager (Velay and Tabeshfar, 1995) transfers data from the computer aided engineering package I-DEAS, which is used by a designer to create a new product or component, to two expert systems. These expert systems then assist the engineer in the material and manufacturing process selection.

If the set of design solutions is not large and the components of the design and their relationships are known, then the same structure can be used for analysis and synthesis (Rosenman *et al.* 1987). This approach is particularly useful for design problems which can be subdivided into independent sub-problems. In both design processes a solution is generated for a problem before it is analysed and evaluated (Oxman and Gero 1987); however, in design analysis the design solution is provided by the user instead of being generated by the expert system.

In the automated process of design synthesis, design is treated as a search through a space of solution states (Oxman and Gero 1987). New design states are developed in a process of analysis and regeneration. Generation is achieved by firing rules where an initial state is transformed to a subsequent or final state by operations initiated by the appropriate rules. Figure 72 shows a graphical representation of the search in a space of solution states.

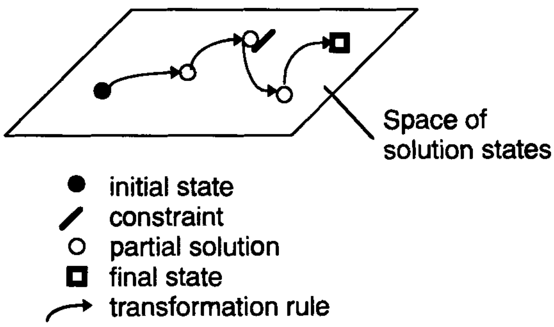


Figure 72: Search in a space of solution states

This state space search is essentially an automated version of the iterative design approach required for thermowells as suggested in section 2.1.

The expert system developed for British Rototherm had to be capable of performing both processes connected with design. Design synthesis will be used to generate a thermowell for a specified application, whilst design analysis (or diagnosis) will be used to verify thermowell designs according to given customer specifications.

First of all, however, it has to be established if thermowell design is a suitable topic for expert system development.

3.2 Suitability for Expert System Development

Developing an expert system is a time consuming process. Even though the idea of using an expert system for a specific task might seem appealing, there might not necessarily be a real need for it, or there are only limited benefits from using it. In other words, expert systems should only be considered if their development "is possible, justified and appropriate" (Waterman 1986, p127). In order to establish the suitability of the given topic of thermowell design for expert system development, a set of guidelines were applied.

3.2.1 Is Expert System Development Possible?

There are several requirements that need consideration when deciding whether the development of an expert system is possible (Waterman 1986):

- The problem to be solved <u>does not require common sense to derive a solution</u>. This knowledge of the world and how it works is unique to human beings and cannot be reproduced sufficiently by a machine. As can be seen from the established design methods, there is no need to apply common sense when designing a thermowell; all design steps are reproducible by a computer.
- The task <u>does not require physical skills</u>. Clearly, the design of thermowells is done without physical manipulation of objects.
- The task is <u>not too difficult</u>. A task that takes an expert days or even weeks to carry out is not suitable for an expert system approach; the same can be said when it is not possible to teach the process to somebody else as expertise can *only* be developed through experience.
- It is also important <u>that the task is well understood</u>. A task that requires research to find a solution is not suitable for expert system development. This should not be mistaken with research that is carried out to establish methods and procedures that will be used by the expert system, as was done in this project.

These characteristics deal with the task or problem at hand. However, there are also requirements dealing with the expert.

- <u>Experts have to exist</u>. The review on thermowell related work showed that there are several experts in the field of thermowell design and applications and their knowledge has been extracted from the published papers.
- <u>The experts have to agree on the solutions</u>. It is virtually impossible to validate an expert system if different experts have different opinions on certain aspects of the task. A solution established by the expert system might be correct

according to one expert, but another expert could see it as the wrong solution. In the case of thermowells all the work has been based on the paper by Murdock (1959); there are no disagreements between experts. If other recommendations are made then they only represent refinements or changes for specific cases which still can be solved with the standard approach.

As all these points are true for thermowell design it can be concluded that expert system development is possible.

3.2.2 Is Expert System Development Justified?

The development of an expert system for a particular task has to be justified; being able to develop an expert system does not automatically mean it is desirable to do so. According to Waterman (1986) there are several ways of justifying the development, some of which are appropriate for thermowell design:

- <u>Task solution offers financial benefits</u>. British Rototherm expects an increased demand for thermowells, therefore the development is justified from a commercial point of view.
- <u>Human expertise is being lost or is scarce</u>. As was mentioned earlier, British Rototherm lacks mechanical design expertise. At the end of the Teaching Company Scheme there will be a lack of expertise, time and manpower to carry out routine thermowell design. To compensate for this lack the development of an expert system is justified.

Clearly, the development of an expert system for thermowell design is justified in more than one way.

3.2.3 Is Expert System Development Appropriate?

The decision if the development of an expert system is appropriate is influenced by the nature, complexity and scope of the task (Waterman 1986).

- <u>Scope</u>: The task needs to be sufficiently broad to have a practical value and narrow enough to be of manageable size. The practical value of an expert system for thermowell design has already been discussed; the arguments for the justification of the development are all based on the practical value of the expert system. The review of thermowell related work also showed that the considerations that have to be taken into account when designing a thermowell (or verifying a design) are limited. Should the expert system have to deal with the design of a complete process plant, and not only with one component like a thermowell, then the size of the task would not be manageable.

- Complexity: The problem an expert system has to solve should not be too easy. It should take a human considerable time to achieve the status of an expert in the particular field. A simple task not suitable for an expert system would be sorting. However, when looking at the diagram in Figure 73 of the inter-relationships between thermowell properties, process conditions and temperature sensor it can be seen that the design of a thermowell is not straightforward; changing one attribute will influence several others. It requires some expertise to carry out appropriate modifications to achieve a satisfactory design.

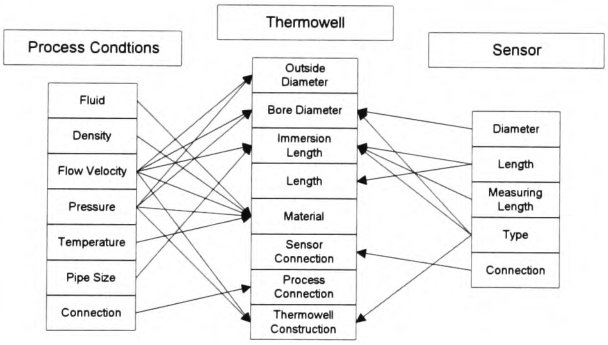


Figure 73: Influence of sensor and process conditions

- Nature: The task should require heuristic solutions; if an algorithm can be used that results in the correct solution every time then expert systems are not appropriate. For the thermowell design rules of thumb are used to make changes to the design, for example the reduction of the immersion length. There are several ways of reducing the length, e.g. using a half or a third of the current length, but none of these will always be correct. Also, if some information is not given, for instance the bore or sensor diameter, assumptions have to be made about this information in order to carry out the design procedures.

In conclusion it can be said that the development of an expert system for thermowell design is possible, justified and appropriate, thus fulfilling all three criteria that decide about the suitability of an expert system for a given task.

3.3 Selection of the Building Tool

There is a wide range of commercial building tools for expert systems available, none of which were developed to handle specific problems. In order to build a tool that suits a given task, the developer of the tool has to know which features are required for the problem to be solved. Considering that there is an unlimited number of possible problems an expert system might have to deal with, it cannot be expected that there is a specific tool available for each one of them. The converse, however, is true, as is stated in Davis' law: "For every tool there is a task perfectly suited for it" (Waterman 1986, p142). The reason for this is simply that one combination of features provides a good tool for one problem and a different set of features is suitable for another problem.

Because of this variety of shells the designer of an expert system has to make a conscious decision about the building tool. A good guide when deciding which tool to use is to consider the following questions (Waterman 1986):

1) Does the tool provide the developer with the power and sophistication needed?

   The available tools can be separated into three types: programming languages (e.g. C++), knowledge engineering languages (e.g. KEE) and application development tools (e.g. KAPPA-PC) which are a combination of the previous two, usually with a non-standard programming language.

   The most flexible tool is a programming language and knowledge engineering languages are the least flexible; the flexibility of application development tools is in between the other two.

   Due to the lack of experience with suitable programming languages, they have to be removed from the list of possible tools that can be used. Furthermore, the expert system has to be used by staff of British Rototherm; this means the expert system has to be designed to specific requirements from the Company. Therefore it is reasonable to use the more flexible tool, an application development tool, as the expert system shell.

2) Are the tool's support facilities adequate considering the time frame for development?

   Support facilities are features such as debugging aids and the input/output facilities of the system. The presence of appropriate facilities will help during the development of the system and also speeds up the whole development process. If the shell already provides input/output facilities, for example, the developer does not have to spend time on creating those facilities himself but can rather concentrate on the more important task of developing the knowledge-based side of the system.

3) Is the tool reliable?

   A reliable tool is essential for the development of any type of software, not only an expert system. It is difficult to concentrate on the development of an application if it is not clear whether any errors that may occur are caused by the application under development or by the tool used to aid the development. In that case the shell can be more of an obstacle than an aid.

   Tools are likely to be reliable when they have a large user group, have been thoroughly tested and are well-debugged; this can be expected from any commercially available tools.

4) Is the tool maintained?

   Having said that the tool should be reliable it has to be kept in mind that it is almost impossible to debug software a hundred percent. There is a small chance that some circumstances, which have never been encountered during testing, will cause the software to produce an error. In this case it is important that the software provider can be contacted in order to rectify the problem. If an old tool is used it can be very difficult to find somebody willing and able to carry out any maintenance. It is therefore important to pick a tool that is still maintained by the tool's developers.

5) Does the tool have the features suggested by the needs of the problem and the application?

   This is really two questions in one, but because a given problem might require certain problem-solving features which in turn require specific features of the application and vice versa, they cannot be considered independently.

Considering the tight time schedule for this project, a decision upon which shell to use for building the expert system had to be made swiftly yet with certainty. As was said earlier, due to the lack of experience with expert systems an application development tool was the preferred option. There were three likely candidates: KAPPA-PC, M.4 and APPLICATION MANAGER. Demonstration versions of all three packages were obtained to aid the evaluation process.

Table 20 compares features of the three application development systems. Based on these features, the suitability of the system for the development of the application is determined.

Table 20: Features of application development systems

Development tool / Feature	KAPPA-PC	M.4	APPLICATION MANAGER
support facilities	yes	yes	yes
reliability	yes	yes	yes
maintainability	yes	yes	yes
rule-based reasoning	yes	yes	no
object-oriented	yes	yes	no
certainty factors	no	yes	no
access to other applications	yes	yes	yes
GUI design options	yes	limited	yes
code generation	yes	yes	no
stand-alone application possible	yes	yes	no
suitable for the application	yes	yes	possibly

Both KAPPA-PC and M.4 are quite similar in their structure and their features. KAPPA-PC offers a more flexible facility to create the graphical user interface (GUI) used to communicate with the user, but lacks the certainty factors that can be used with M.4. Both use an object-oriented structure, but KAPPA-PC uses a graphical representation of these objects in an 'object tree', which makes the use of objects more accessible to the non-programmer. APPLICATION MANAGER, on the other hand, is neither object-oriented, nor does it offer any knowledge based features such as forward or backward chaining. Because the latter is important in order to build an expert system, APPLICATION MANAGER has been excluded from the selection process.

The use of certainty factors with rules, as offered by M.4, is not essential for thermowell design; this is more appropriate for applications dealing with diagnostics where uncertainties are present. For example, certainty factors are used in a system called MYCIN, which diagnoses bacterial infections in hospital patients (Waterman 1986, p34). However, this is not a reason to exclude M.4 because these certainty factors do not have to be used in order to develop an application. Nevertheless, the more flexible GUI design options offered by KAPPA-PC, together with the fact that it has been used in the University's School of Design and Advanced Technology successfully before and the existence of a mailbase (a facility that allows communication with other users and the developers of KAPPA-PC through the Internet) suggested that KAPPA-PC was the preferred choice for this project. A short introduction to KAPPA-PC can be found in Appendix V.

3.4 Systematic Approach for Software Design

Programming any kind of software, including expert systems, is a form of design. It is therefore reasonable to adopt a systematic design approach for the development of the expert system similar to the one described for thermowells.

The development of the expert system is original design, according to the definition given in the chapter 2.3 *Systematic Design Approach*. The application of the design process, as described in the same chapter, is not as straightforward, however. The first step, the clarification of the task, is an important requirement for any design task and should not be neglected. However, in this particular case the conceptual and embodiment design phases cannot be considered in too much detail because of the time restrictions involved in the project. It is therefore not possible to establish different concepts of how the expert system could be implemented, and from this decide on the most suitable solution.

This design approach can be compared with an approach proposed specifically for program design (Robertson 1991, p2-3). This approach consists of seven steps:

1) define the problem
2) outline the solution
3) develop the outline into an algorithm
4) test algorithm for correctness
5) code the algorithm into a specific programming language
6) run program on a computer
7) document and maintain the software

Step 1) in this approach is equivalent to the determination of the design specifications. Steps 2) to 4) are similar to the conceptual and embodiment design phases. Similar to mechanical design, it is possible in software design that there are several solutions to solving one problem. The solutions have to be compared and evaluated, and the most suitable solution is chosen for implementation. Steps 5) and 6) deal with the detail design, or implementation, of the chosen solution. The final step 7) is not unique to software design; in mechanical design it is the design review and improvement once a product is in production.

Both approaches indicate that it is important to establish the specifications for the expert system before the development can begin. Table 21 details the design specifications for the expert system. D and W denote whether the requirement is a demand or wish, respectively (Pahl and Beitz 1988).

Table 21: Requirements for expert system

Requirements	**D/W**
Functionality:	
analysis of all three types of thermowells	D
synthesis of all three types of thermowells	D
analysis/synthesis to satisfy vibration criterion	D
pressure criterion	D
stress criterion	D
tapered thermowells according to ASME PTC19.3	D
determination of thermal effect	W
estimation of thermowell price	W
synthesis to suit specified sensor	D
material and fluid properties stored in database	D
User interface:	
data input using different units (metric or imperial)	D
input of flow rate instead of flow velocity possible	D
load/save feature	W
print facility	D
modification of existing calculations possible (with indication)	W
help facility	W
results in tabulated form	W
restricted access to modify material and fluid properties	D

These specifications form the basis of the expert system development.

3.5 Structure of the Expert System

The object oriented structure of KAPPA-PC (see Appendix V) suggests that the expert system should be organised according to the different objects involved in the thermowell analysis and synthesis, such as the thermowell itself, the customer and the materials that are available. This structure can be combined with the rule-based features of KAPPA-PC to implement the design procedures. Figure 74 shows the object hierarchy of the expert system as it was originally proposed. Note that the instances of the classes **Materials** and **Fluids** are only examples; there are many more materials and fluids available which are not shown.

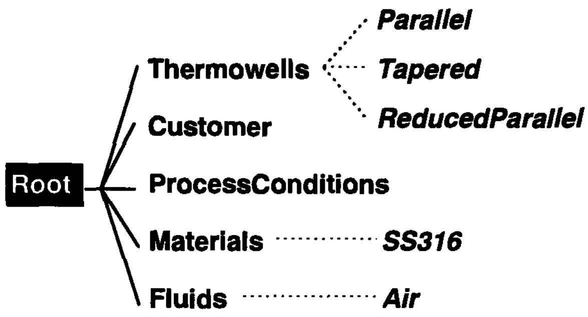


Figure 74: Initial structure of the expert system

The class **Customer** is used to define the customer name, customer order number and the internal (Rototherm) works number. In the class **ProcessConditions** the different process conditions such as temperature and pressure are stored. Instances of different materials and fluids are defined under the classes **Materials** and **Fluids**, respectively. Each instance contains the information concerning the associated material or fluid. For example, the instance ***SS316*** contains the tables for the modulus of elasticity, specific weight, specific heat and thermal conductivity at various temperatures for 316 stainless steel. It also contains information about the price of various bar size of the material, what maximum size is available and if the material is available as solid bar and tubing. The instances of class **Thermowells** would contain the information defining the appropriate thermowell, e.g. immersion length, bore diameter, root and tip diameter (in instance ***Tapered*** only), reduced diameter and length (in instance ***ReducedParallel*** only), etc.

However, during the process of implementing the thermowell analysis part of the application it was soon noted that this structure has its disadvantages. The fact that the thermowell information was stored in the appropriate **Thermowells** instances meant that only one thermowell of each type could be stored at the same time. This is not acceptable because it is quite common for a client to order several thermowells of one type. Therefore, it was decided to add a sub-class **Order** to the class **Customer**. For each thermowell of the current order an instance will then be created under that sub-class, storing all the relevant information to carry out the analysis. To identify the thermowells, the tag-number of the individual thermowells is used as the instance name. See Figure 75 for the added sub-class and an example for the instances.

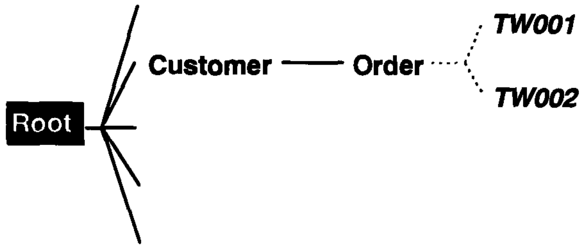


Figure 75: New structure to deal with different thermowells

This structure proved to be sufficient for the initial thermowell analysis procedure which did not include the pricing option (this option was added later during the project). The information entered by the user is first stored in the classes **Customer**, **ProcessConditions** and **Thermowells**. The analysis for each thermowell is carried out after all the necessary data has been entered. Therefore, before the analysis is carried out, the information specified by the user is moved to the appropriate **Order** instance (for example, ***TW001***, see Figure 75). This enables the information to be saved in the same format as specified by the user. Only then will the unit conversion be carried out. The conversion is necessary because the correct units have to be used for the equations. The results of the analysis are also stored in the appropriate instance. This way, both the results and the original information can be saved to disk.

To include the pricing option and carry out thermowell synthesis it was necessary to modify the proposed structure again by adding more objects. In order to carry out the design synthesis, information about the sensor, the process connection and the available tubes is required. Therefore, appropriate sub-classes were added to the newly created **Design** class; see Figure 76. Also, a sub-class **DesignedThermowell** was added which will contain values required during the design synthesis process, for example the minimum and maximum lengths *minL* and *maxL*, which can be specified by the user. For each thermowell that has to be designed, an instance under this class is created which is used to store the information as specified by the user. The name of the instance is the tag number of the thermowell together with the prefix 'D_'; see Figure 76 for an example.

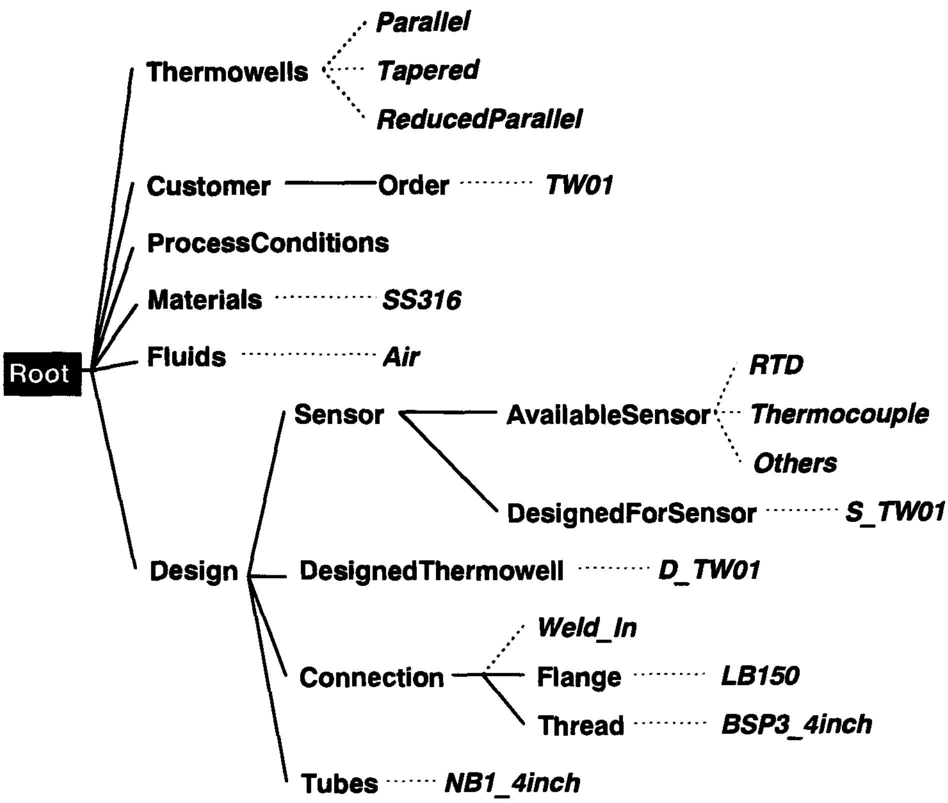


Figure 76: Final structure implemented in the expert system

The instances of class **Tubes** contains the information about the individual tube sizes (in Figure 76 only one instance, ***NB1_4inch***, is given), namely the outside diameter (constant for each tube size) and the wall thickness (which changes with changing schedule and is different for each size). Note that in the case of instance ***NB1_4inch***, 'NB' stands for nominal bore and the '1_4' actually represents ¼. Due to restrictions imposed by KAPPA-PC, a class or instance names cannot begin with a number and special characters such as a slash '/' cannot be used, either. Only the character '_' is permissible.

Similarly, the sub-classes **Flange** and **Thread** of class **Connection** contain the details about the different connections. Again, the two instances ***LB150*** and ***BSP3_4inch*** in Figure 76 are only examples, and more connections are available in the expert system. In the case of a weld-in socket, no selection has to be made as the user can directly specify the diameter of the socket, or the expert system will determine the diameter from the dimensions of the designed thermowell. The instance ***Weld_In*** is used to store the value of the socket's diameter and make it possible for the user to select this type of connection.

The sub-class **AvailableSensors** and its instances ***RTD***, ***Thermocouple*** and ***Others*** is purely used to allow the user to select a sensor type. The properties describing the selected sensor, for example the stem diameter, are stored in an instance created under the sub-class **DesignedForSensor**. The name of the instance is the tag number of the thermowell together with the prefix 'S_'; see Figure 76 for an example. This instance is only created if a sensor has been specified by the user.

These classes and instances represent the objects dealing with the thermowell analysis and synthesis. However, in order for the expert system to work it is necessary to create additional objects and modify existing ones, see Figure 77. The instance ***Global***, which is present in every application developed using KAPPA-PC, is used to store temporary values and information such as whether synthesis or analysis is carried out, or the value of constants such as π used in the application.

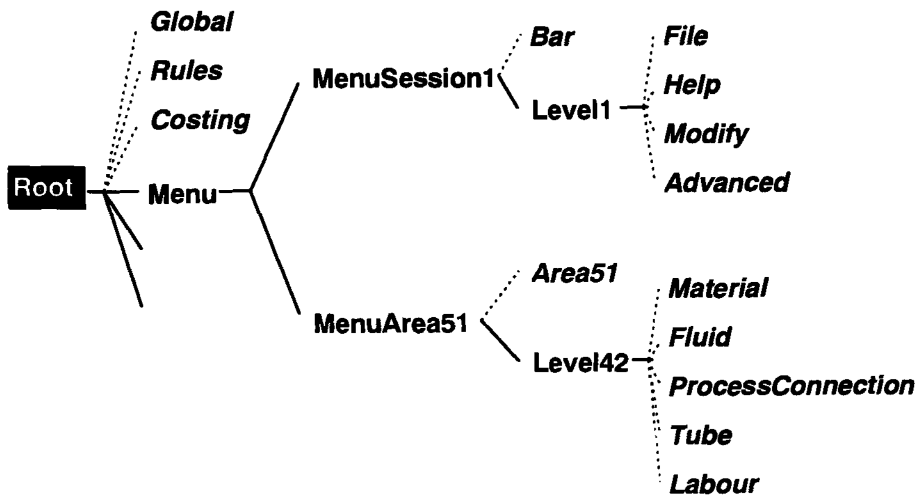


Figure 77: Additional objects for the application

The instance ***Rules*** contains the different rule sets which are used throughout the design synthesis process. Each slot of the instance contains a number of rule names which are used by the inference engine at the appropriate time. The name of the slot identifies the rule set. For example, the slot **Connections** contains the rule names **Thread, Weld, Flange, CheckProcessConnection** and **ProcessConnection**. When this rule set is called by the inference engine the rules with the appropriate names will be fired, provided their IF part is true. The rules **Thread, Weld** and **Flange** establish which of the three types of connection is suitable for the application. The rules **ProcessConnection** establishes the appropriate size for the connection and rule

CheckProcessConnection compares the connection size with the limitations specified for that size.

The instance ***Costing*** contains some additional information that is necessary in order to estimate the selling price for a thermowell. This information is only temporary, but unlike the values in instance ***Global*** it is not necessary for the general operation of the application but for the costing step in particular. Therefore, this additional instance was created.

Finally, a few sub-classes and instances had to be created under the class **Menu**. These are required to specify the options and appropriate functions of the custom menu bars used in the application.

It has to be noted that every session window, button, input field etc. is an object in KAPPA-PC and is therefore represented as an instance in the hierarchy. However, these objects are created automatically by KAPPA-PC and are therefore not explained in this section. Furthermore, the application in its current state contains 50 classes and 224 instances, which makes a detailed description undesirable.

3.6 Shared Features of Thermowell Analysis and Synthesis

Even though the goals to be achieved with the two processes are different - analysis verifies an existing design, synthesis creates a new design for a given application- a number of features can be used for both processes. These features are discussed in the following sections.

3.6.1 Functions for Thermowell Analysis and Synthesis

Because the same equations are used in both the thermowell analysis and synthesis process, the same functions can be used to carry out the calculations. The functions are designed in such a way that the values necessary for the calculation are passed on to the function as the arguments of the function call. It is therefore possible to call the function without setting specific slot values first. This is especially useful when the functions have to be tested. For example, the function Frequency(y) is used to calculate the natural frequency of a thermowell, based on equation [12].

The equation is the same for every type of thermowell and independent of whether analysis or synthesis is carried out. The only variable is the maximum deflection y_{max}, which is passed on to the function as the argument y in the function call Frequency(y). Similarly, there is a function for each thermowell type that calculates the maximum deflection of the respective thermowell. Other shared functions are: Constants() - calculation of K_a, F_A and F_B; MoreConstants() - calculation of K_x, K_1, K_2, K_3; WakeFrequency() - determines the wake frequency f_w; Ratio() - establishes the frequency ratio r.

3.6.2 Creation of a Thermowell Instance

This feature is necessary to enable a save-to-disk facility. All the information dealing with a thermowell to be analysed or that has been designed is stored in an instance with the thermowell's tag-number as its name under the class **Order**. The instance is created by the function MakeANewOne().

After the instance has been created, the relevant information has to be copied into the instance. This is done with the function MoveIt(). This function is also used to copy the information into the class **Order** or when a saved calculation is loaded from disk and has to be displayed. The instances are used to store the specifications in the form the user entered them, i.e. with the same number of digits or the original units. This way, thermowells can be identified more easily by the client. The unit conversion and the actual calculations are carried out using the values in the **Order** class. The results of the calculations are then copied into the appropriate instance so that they can be saved together with the original information.

3.6.3 Unit Conversion

The calculations are carried out using units of metres, seconds and kilograms. However, in order to make the data input more convenient for the user, the design and thermowell specifications can be entered using different units, such as inches for the length or fts^{-1} for the flow velocity. It is therefore necessary to convert the units specified by the user into units that can be used for the calculations. To make this conversion possible, it is necessary to provide the user with a selection of units that can be used. For that reason ***ComboBox*** images are used in the application which display the available units.

As discussed in the section *Structure of the Expert System* the relevant information is stored in slots in the appropriate objects. Each characteristic that has a value and a unit has two slots; one for the actual value, the other for its unit. For example, the length of a thermowell is stored in the slots **Thermowells:Length** and **Thermowells:DimLength**. The functions carrying out the conversion check all relevant slots containing units. If the units are not the units required for the calculations, a conversion factor is applied to the appropriate slots containing the values for the characteristics and the units are changed to the correct units. For instance, if an immersion length of 150mm has been specified, a conversion factor of $^1/_{1000}$ is applied, resulting in 0.150m. The conversion is only carried out in the slots of the class **Order**. The original information is stored in the appropriate instance of class **Order**.Creation of a Thermowell Instance

3.6.4 Calculation of Flow Velocity from Flow Rate

The flow velocity is required in order to calculate the wake frequency caused by the fluid flow. In some cases the client specifies the flow rate and not the flow velocity. It is therefore necessary to determine the velocity from the specified flow rate. Flow rates are commonly specified in 'volume per time' or 'mass per time', e.g. m^3hr^{-1}, kgs^{-1}, lbs^{-1}, etc. In both cases, the diameter of the pipe the fluid is flowing through has to be specified by the client, too. The flow velocity can then be calculated by dividing the flow rate by the appropriate pipe cross-sectional area, i.e. $v = {}^V/_A$ with $A = {}^\pi/_4 d^2$.

If the flow rate is given in 'mass per time' then the flow is determined in 'volume per time' first. This is done by multiplying the given flow rate with the specific volume of the fluid, specified in m^3kg^{-1}. The specific volume of the available fluids are stored in the expert system's database. The velocity can then be established with the same equation as above, $v = {}^V/_A$.

A conversion of units has to take place as well. It is necessary when the flow rate is not given in either m^3s^{-1} or kgs^{-1} and is carried out as described in section *Unit Conversion*.

The data input window for the analysis and synthesis has a ***Text***, ***Edit*** and ***ComboBox*** image for both the flow velocity and the flow rate. It is the choice of the user which property will be used in case the flow rate and velocity are given. Should the user enter both values, then the expert system will use the directly specified flow velocity instead of the flow rate. If the flow rate is entered, additional ***Text***, ***Edit*** and ***ComboBox*** images are displayed which allow the user to enter the pipe diameter and its units.

3.6.5 Determination of Material and Fluid Properties

There are several material and fluid properties which are required by the calculations used for the analysis and synthesis. These properties have to be at the given process temperature. The material properties are the modulus of elasticity E, the specific weight G and the maximum allowable stress S. In case the temperature error also has to be established the specific heat C_p and thermal conductivity k of the material are required, too. The fluid property that has to be established is the fluid density.

The values for these properties are stored as lookup tables in the materials database and are given for a range of temperatures. They have to be interpolated for the given process temperature. Table 22 represents the lookup table for the modulus of elasticity for 316 stainless steel.

Table 22: Lookup table for the modulus of elasticity

Temperature [°C]	Modulus of elasticity [$\cdot 10^9$ Nm^{-2}]
20	201
100	193
200	184
300	175
400	165
500	158
600	148
700	141
800	133

If the process temperature is equal to a temperature given in the lookup table, then the modulus of elasticity can be directly taken from the table; otherwise, linear interpolation is carried out. This method is used for all the material and fluid properties that have to be established and is carried out by a function called InterpolateValues().

3.6.6 Thermal Considerations

This part of the expert system in its present state is used purely to provide the client with information about the thermowell's measurement performance. When called during the thermowell synthesis process, this procedure will not trigger any changes to the thermowell geometry. The relevant characteristics determined are the temperature difference between process fluid and sensing element and the time constant of the thermowell (Richmond 1980). As was discussed earlier, some additional values have to be entered by the user at this point to carry out the calculations. These values are the length L_1 of the thermowell that is completely immersed in the fluid, the overall heat coefficient U and the ambient temperature T_a. Once this information has been specified the thermal characteristics of the thermowell can be calculated.

The temperature error, i.e. the difference in temperature between the fluid and the temperature sensor, is determined using the equation

$$T_e = \frac{T_f - T_a}{\cosh(ML_1)}$$

with $M = \sqrt{\frac{\pi DU}{ka}}$: heat transfer factor

D: outside diameter of thermowell

U: overall heat coefficient

k: thermal conductivity of thermowell material

a: cross-sectional area of thermowell

L_1: length completely immersed in fluid

T_f: temperature of process fluid

T_a: ambient temperature

The outside diameter D for tapered thermowells is the thermowell's mean outside diameter, i.e. $D={}^{A-B}/_{2}$. For reduced parallel thermowells the reduced diameter is used. The cross-sectional area is calculated using the outside diameter D.

Generally, the length L_1 is the same as the immersion length L. However, in cases such as illustrated in the chapter on *Design Specifications* (restrictions for thermowell length), the immersed length L_1 can be shorter than the immersion length L.

For the calculation of the time constant the equation

$$\tau = \frac{mC_p}{UA}$$

with m: mass of thermowell

C_p: specific heat of thermowell material

A: area of thermowell surface

is used. The mass and surface area of the thermowell can be calculated either from the information about the thermowell material and geometry provided by the user (thermowell analysis) or from the design that has been produced by the expert system (thermowell design). The specific heat and the thermal conductivity of the thermowell material is specified in the expert system's material database. In case a material has been added to the database without specifying these properties, the expert system will prompt the user to specify the missing information at this stage.

3.6.7 Estimation of Thermowell Price

The price of a specific thermowell is determined using standard British Rototherm procedures. The basic manufacturing cost of a thermowell is the sum of the material cost and the labour cost, i.e. $C_{manuf} = C_{material} + C_{labour}$. A percentage P is added to the manufacturing cost to give the minimum selling price, i.e. $Price = C_{manuf} \bullet (1 + {}^{P}/_{100})$. (For reasons of confidentiality no values can be given for the percentage P, any other mark-up values, the machining times, labour cost or material prices.)

The material cost $C_{material}$ is the sum of the following individual costs:

- cost of raw material required to manufacture a thermowell of given diameters and length
- cost of flange, if required
- cost of raw material necessary to manufacture lagging extension, if required

The length of the thermowell is not just the immersion length; it also includes the length of the threaded connection and the hexagonal adapter, the weld-in socket or the specified flange.

The raw material for the lagging extension is assumed to be the same material and diameter as for the thermowell.

The labour cost C_{labour} is calculated by establishing the labour time T_{labour} and multiplying it by the hourly rate $^{£}/_{hr}$, i.e. $C_{labour} = T_{labour} \bullet ^{£}/_{hr}$. The labour time depends on various aspects of thermowell manufacture:

- profiling time for solid thermowells $T_{profile}$

  This refers to parting off the raw material, turning down the diameter to the appropriate dimensions over the immersion length and machining of the thread or weld-in socket.
- gun drilling time for solid thermowells T_{drill}

  Time required to drill the bore of the thermowell. The overall length, including the lagging extension, is used for this process.
- machining time of flange T_{flange}

  Only required when a flange is used as the process connection. Machining of the flange is necessary to drill a bore that suits the thermowell.
- machining time of threaded connection T_{thread}

  This time is only applicable for fabricated thermowells with a threaded connection. Common sizes of threaded adapters, which have to be welded to the tube, are manufactured on a regular basis and held in stock. They are not manufactured specifically for an order, unless unusual types and sizes are required. This time therefore represents an average machining time, independent of type and size.
- welding time for fabricated thermowells $T_{weld,thermowell}$

  Welding the tube, end plate and process connection (except flange). The time includes the time for cutting the tube to length.
- welding time for flanged process connection $T_{weld,flange}$

  The time depends on the type of weldment (fillet or full penetration weld) used.

Each time includes the appropriate loading/unloading and set-up time. Therefore, the labour times can be calculated as follows:

- solid thermowell with threaded connection or weld-in socket:

  $T_{\text{labour}} = T_{\text{profile}} + T_{\text{drill}}$
- solid thermowell with flanged connection:

  $T_{\text{labour}} = T_{\text{profile}} + T_{\text{drill}} + T_{\text{flange}} + T_{\text{weld,flange}}$
- fabricated thermowell with threaded connection or weld-in socket:

  $T_{\text{labour}} = T_{\text{weld,thermowell}} + T_{\text{thread}}$
- fabricated thermowell with flanged connection:

  $T_{\text{labour}} = T_{\text{weld,thermowell}} + T_{\text{flange}} + T_{\text{weld,flange}}$

Applying the above cost relationships the minimum selling price for a particular thermowell can be established.

3.7 Thermowell Analysis

The thermowell analysis part of the expert system is used to check whether an existing design from an earlier order or provided by the customer passes the 'wake frequency analysis'. This term is commonly used in industry and usually refers to the vibration, pressure and stress analysis. Additionally, the expert system has been designed to carry out an estimation of the temperature error introduced by the thermowell and a calculation of the thermowell price, if this information is required.

This part of the expert system was implemented first, because the analysis of a thermowell can be carried out simply by applying the available equations without making any changes to or assumptions about the thermowell's geometry or the process conditions. This simplifies the development of the thermowell analysis feature considerably, which is important considering the lack of experience using KAPPA-PC or developing an expert system.

Figure 78 shows the general procedure used for thermowell analysis. First, the user has to specify the information required for the analysis; then the analysis will be carried out by the expert system. If another thermowell has to be analysed for the same customer, the procedure of data input and analysis will be repeated. Otherwise the results of the analysis will be displayed and the analysis procedure has been completed.

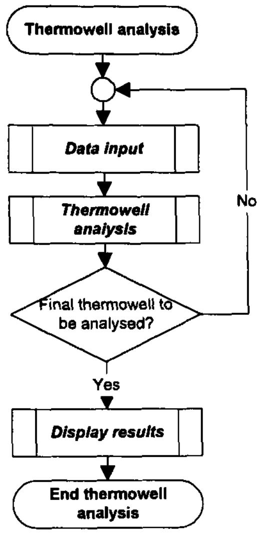


Figure 78: Thermowell analysis procedure

3.7.1 Data Input

The first step in implementing the analysis procedures was to create an interface that allows the user to specify all the necessary information about the thermowell and process in order to carry out the analysis, together with the information concerning the customer and order.

3.7.1.1 Graphical User Interface for Data Input

KAPPA-PC provides facilities to create graphical user interfaces. Therefore, a window ***Session1*** was created, together with the according ***Text***, ***Edit*** and ***ComboBox*** images. The ***Text*** images explain which information is required, e.g. the bore diameter, and the ***Edit*** images are used to enter the appropriate value, e.g. 12. The ***ComboBox*** images have several functions: they are used to specify the dimension for the given value, e.g. mm, and they are also used to select the thermowell type, material and process fluid. Figure 79 shows part of this window, with the ***ComboBox*** for the thermowell type displaying the available choices.

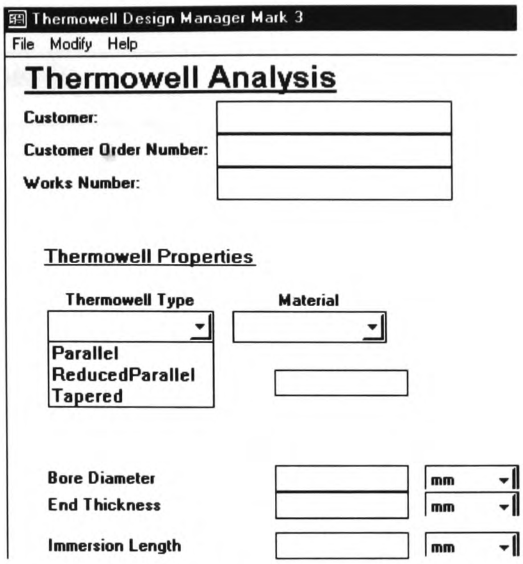


Figure 79: Selection of thermowell type

The ***Text*** image hidden by the open ***ComboBox*** refers to the outside diameter of the thermowell. Depending on the type of thermowell selected, additional images will be displayed. For example, if a reduced parallel thermowell is selected, a ***Text*** image 'Reduced length' will be displayed beneath the image 'Immersion length' and another ***Text*** image 'Reduced diameter' beneath the image 'Outside diameter', together with the appropriate ***Edit*** and ***ComboBox*** images.

Once all the information has been entered by the user, the analysis has to be carried out. To begin this process, a ***Button*** image 'Calculate' was added which will start the analysis. Alternatively, the user might wish to enter the specifications of another thermowell, in case more than one thermowell has to be analysed for one client. Therefore, buttons 'Next' and 'Previous' were added to the window. To analyse another thermowell, the button 'Next' has to be selected instead of button 'Calculate'. The input fields will now be cleared and another thermowell can be specified. Also, the text of the ***Button*** images changes from 'Calculate' to 'Done' to indicate to the user that the data of several thermowells can be entered. The button 'Previous' can be used to step backwards through the already entered thermowells and make corrections in case incorrect information was specified. If all thermowells have been specified selecting button 'Done' will start the analysis and display the results in a separate window in tabulated form. Figure 80 shows the complete window for thermowell analysis.

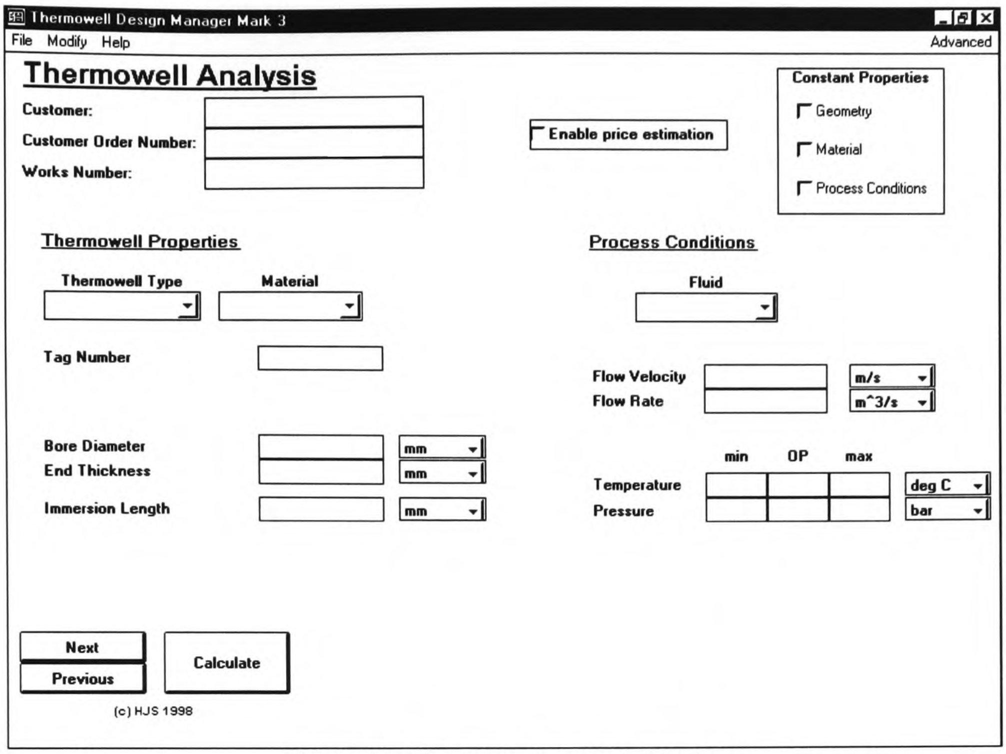


Figure 80: Thermowell analysis window

Two ***CheckBox*** images were added as well. The set under the heading 'Constant Properties' was created to make the data input easier for the user. In case several thermowells have to be analysed for one client it is likely that some of the properties are constant or vary only slightly for several thermowells. Using this feature the necessary data input can be minimised. The second ***CheckBox*** image 'Enable price estimation' indicates whether the price of the thermowell should be estimated.

3.7.1.2 Procedure for the Data Input

Figure 81 shows the procedure concerning the data input for thermowell analysis.

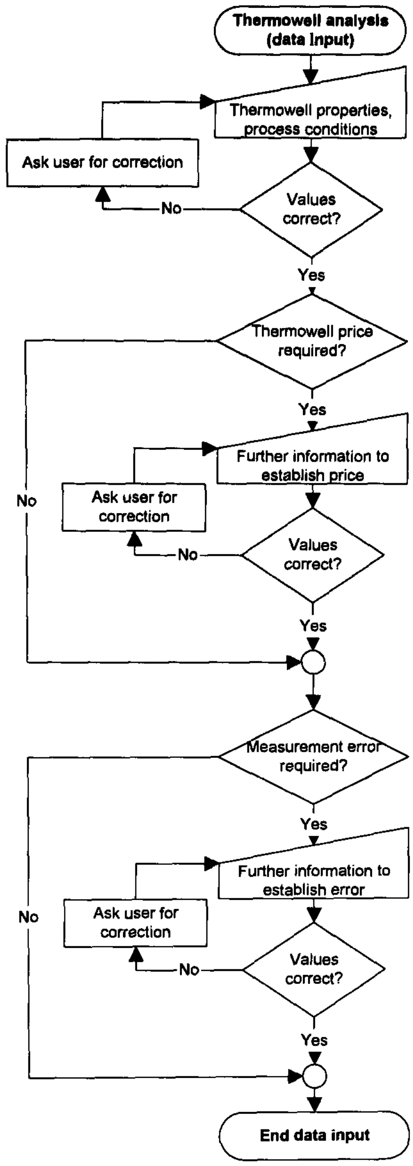


Figure 81: Thermowell analysis

To begin with, the user specifies the thermowell type, material and geometry, and the process conditions. Following this the expert system checks if the values entered are valid. For example, it is only possible to enter numerical values; the root diameter of a tapered thermowell cannot be smaller than the tip diameter; the bore diameter cannot be larger then either outside diameter; etc. It is also checked if all necessary values have actually been specified. If the flow rate is specified instead of the flow velocity, the diameter of the vessel must be specified, too. If any of the values are missing or incorrect, the user will be asked to correct them.

If an estimation of the thermowell price is required, the expert system will ask for more characteristics to be entered:

- the lagging extension which might be required
- the type and size of the process connection
- the size of the sensor connection (the connection between the sensor and the thermowell)
- the type of weldment used for the flange if that is the chosen process connection
- the type of construction (solid of fabricated) if a parallel thermowell has been specified
- the price of the material of the appropriate size in case it is not in the database (this can be the case if a bar size is necessary that is not commonly used)

The estimation of the measurement error also requires additional information:

- the length of the thermowell completely immersed in the fluid
- the overall heat coefficient
- the ambient temperature, i.e. the temperature outside the vessel

The additional information necessary for the estimation of the temperature error and price of the thermowell will also be checked for its correctness.

After all necessary characteristics have been specified, the analysis will be carried out. If the user has chosen to analyse another thermowell, the data input procedure will be repeated; otherwise, this phase is finished.

3.7.2 Analysis of the Thermowell

3.7.2.1 Establishing of the Material Properties

Before the actual analysis is carried out, the properties of the thermowell material at the given process temperature need to be established. These properties are the modulus of elasticity E, specific weight G and maximum allowable stress S. If the estimation of the temperature error is also required, then the thermal conductivity k and specific heat C_p are determined as well. The values for these properties are stored in the materials database for a range of temperatures and have to be interpolated for the given process temperature. See section *Shared Features of Thermowell Analysis and Synthesis* for this procedure.

3.7.2.2 Analysis Procedure

The analysis of the thermowell is outlined in Figure 82.

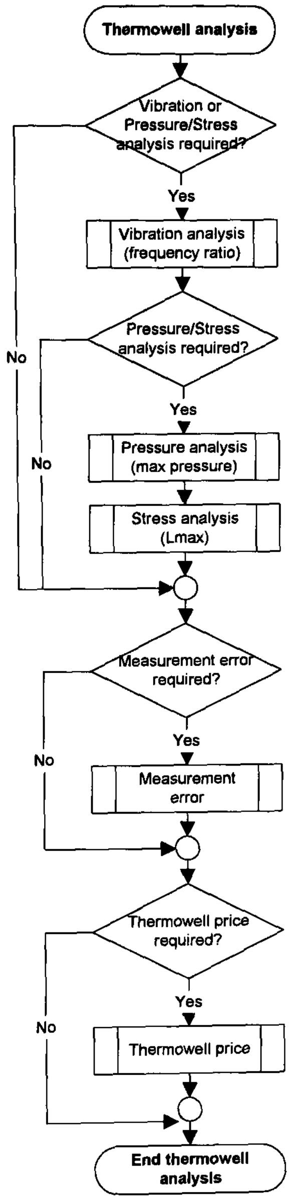


Figure 82: Outline of analysis procedure

3.7.2.2.1 Vibration Analysis (Frequency Ratio)

If the user requires a vibration or pressure and stress analysis or both, i.e. the complete 'wake frequency analysis', then the vibration analysis will be carried out as the first step; see Figure 83 for the vibration analysis process.

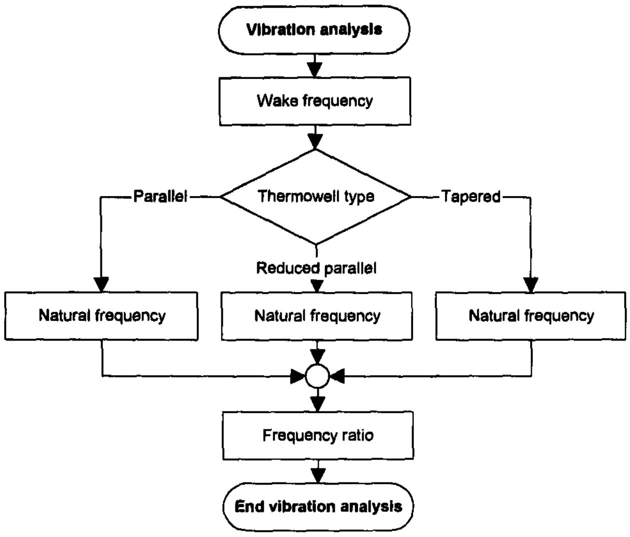


Figure 83: Vibration analysis

First, the wake frequency will be calculated using the established relationship $f_w=0.22^{v}/_{B}$. The natural frequency is determined using the appropriate approach for the type of thermowell. Finally, the frequency ratio r can be calculated and the vibration analysis has been completed.

3.7.2.2.2 Pressure Analysis (Allowable Pressure)

Next, the pressure analysis is carried out by determining the stress constant K_1 and calculating the maximum allowable pressure using the equation $P = K_1S$. To pass this analysis, the calculated maximum allowable pressure has to be larger than the specified operating pressure, i.e. $P > P_O$.

3.7.2.2.3 Stress Analysis (L_{max})

The last phase of the 'wake frequency analysis' is the calculation of the maximum allowable length for the thermowell in the given service conditions. Figure 84 illustrates the individual steps necessary when establishing the maximum length.

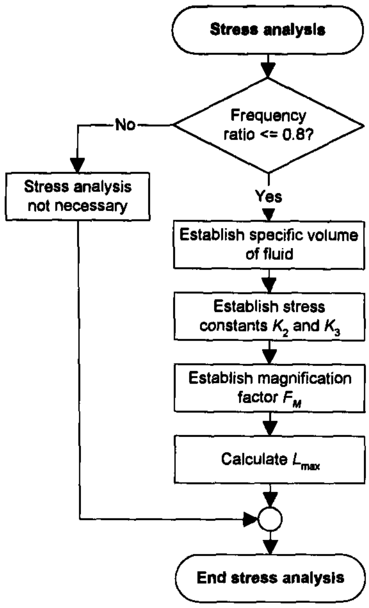


Figure 84: Stress analysis

Before calculating $L_{\max}$ or establishing any of the necessary values for its calculation, it is necessary to check whether the thermowell passed the vibration analysis for two reasons:

- if the thermowell has failed the vibration analysis then there is no need to know the maximum length possible for the thermowell as it cannot be used anyway.
- if $r > 1$, the square root of the equation used for calculation of $L_{\max}$ will be negative; if $r = 1$ the equation used to calculate the magnification factor F_M will cause a 'division by zero' error. In either case, the expert system would cause an error. Therefore, it is best practice to only apply those two equations if the frequency ratio is satisfactory.

After it has been established that the maximum allowable length is required, the specific volume of the fluid at the given operating temperature and pressure is calculated, together with the stress constants K_2 and K_3 and the magnification factor F_M. Using those values, the previously established allowable stress and the given operating pressure the maximum allowable length of the thermowell can be calculated.

This concludes the 'wake frequency analysis'. If the user has not selected to establish the temperature error introduced by the thermowell or the thermowell's price (or both) then the analysis phase has finished. Otherwise the specified additional information will be calculated.

The estimation of the measurement error and the thermowell price are carried out in the same way as was discussed in the appropriate sections in the section 3.6.

3.7.3 Presentation of Results

Depending on whether there is another thermowell to be analysed for the same customer, the user can enter the appropriate information or view the results. If only one thermowell has been analysed, the results will appear on the same screen where the thermowell and application information was specified, see Figure 85. In the case of two or more thermowells, the results will be displayed in tabulated form, see Figure 86. All the information can now be printed out or saved to disk and a new set of thermowell analyses can be carried out.

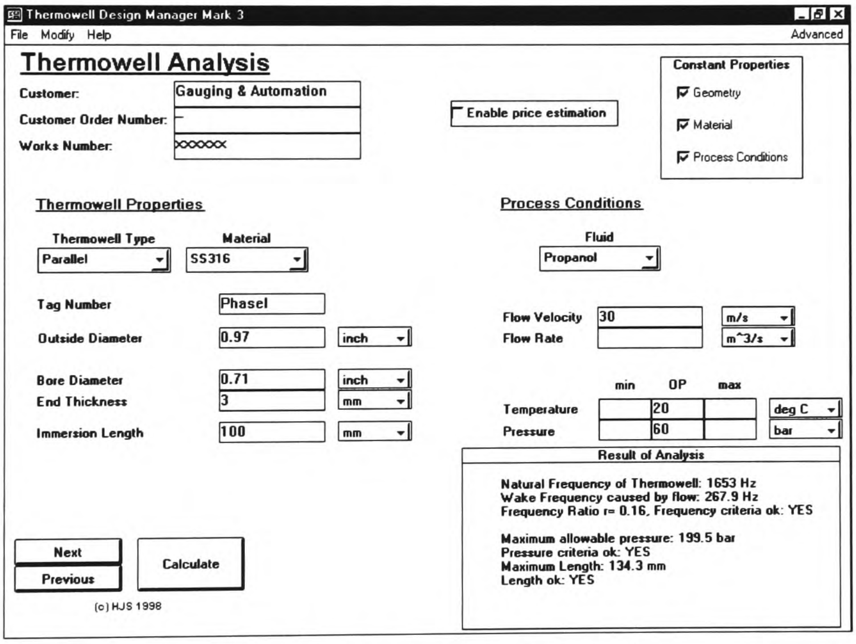
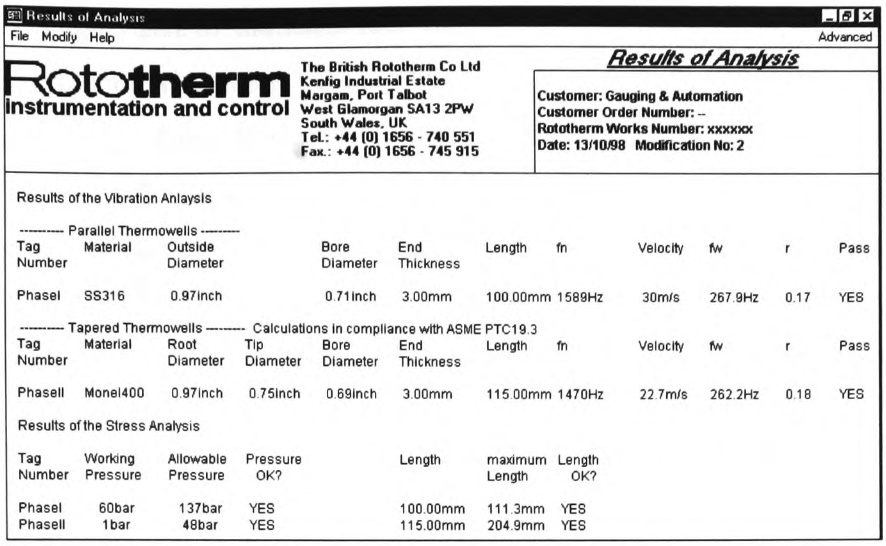


Figure 85: Results for one thermowell



Results of Analysis

File Modify Help Advanced

Rototherm
Instrumentation and control

The British Rototherm Co Ltd
Kenfig Industrial Estate
Margam, Port Talbot
West Glamorgan SA13 2PW
South Wales, UK
Tel.: +44 (0) 1656 - 740 551
Fax.: +44 (0) 1656 - 745 915

Results of Analysis

Customer: Gauging & Automation
Customer Order Number: –
Rototherm Works Number: xxxxxx
Date: 13/10/98 Modification No: 2

Results of the Vibration Anlaysis

--------- Parallel Thermowells ---------

Tag Number	Material	Outside Diameter	Bore Diameter	End Thickness	Length	fn	Velocity	fw	r	Pass
PhaseI	SS316	0.97inch	0.71inch	3.00mm	100.00mm	1589Hz	30m/s	267.9Hz	0.17	YES

--------- Tapered Thermowells --------- Calculations in compliance with ASME PTC19.3

Tag Number	Material	Root Diameter	Tip Diameter	Bore Diameter	End Thickness	Length	fn	Velocity	fw	r	Pass
PhaseII	Monel400	0.97inch	0.75inch	0.69inch	3.00mm	115.00mm	1470Hz	22.7m/s	262.2Hz	0.18	YES

Results of the Stress Analysis

Tag Number	Working Pressure	Allowable Pressure	Pressure OK?	Length	maximum Length	Length OK?
PhaseI	60bar	137bar	YES	100.00mm	111.3mm	YES
PhaseII	1bar	48bar	YES	115.00mm	204.9mm	YES

Figure 86: Results of analysis in tabulated form

3.8 Thermowell Synthesis

Figure 87 outlines the process of design synthesis for thermowells. The following chapters will describe this process in more detail for the different types of thermowells.

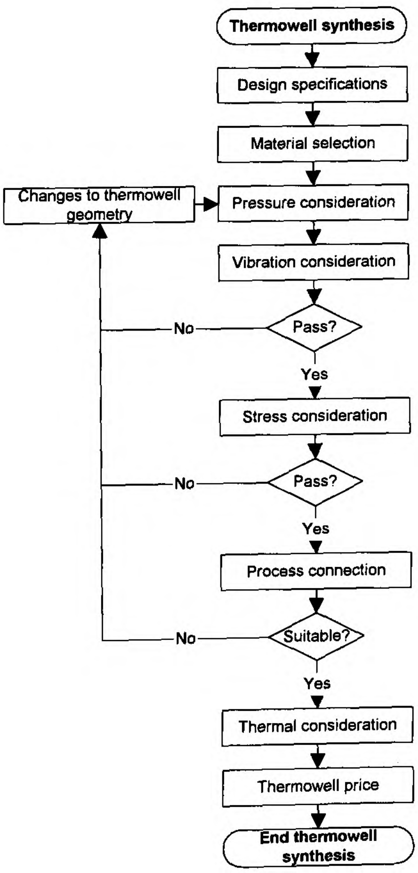


Figure 87: Thermowell synthesis

Even though Figure 87 indicates that the checks for individual design criteria are carried out directly after it has been calculated, it has been implemented in the expert system differently. Figure 88 shows the implemented design synthesis process. There is no change in the order in which the individual design steps are carried out. However, the frequency ratio r, maximum possible length L_{max} and the compatibility between outside diameter and process connection are checked together after those three steps have been carried out and not at the end of the individual design step as implied in Figure 87.

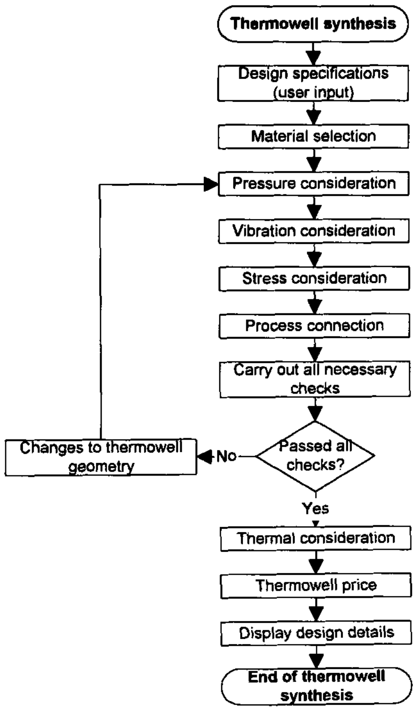


Figure 88: Implemented thermowell synthesis procedure

This was necessary because most changes to the thermowell geometry will influence more than one of the criteria. Consider an example in which the frequency ratio is satisfactory, but the stress criteria is not, i.e. the current thermowell length is larger than the maximum allowable length. Looking at the equation for L_{max}, it can be seen that this length depends on the frequency ratio. Therefore, the frequency ratio has to be calculated for the new length even though the vibration criterion was satisfactory for the previous length. Similarly, should the root diameter of a tapered thermowell conflict with the specified process conditions, the taper has to be changed, i.e. the root diameter is decreased and the tip diameter increased. Usually, a parallel design is used first in such a case. This requires the re-calculation and check of the pressure, frequency and stress criteria. It was therefore more convenient to implement the procedure in the form suggested in Figure 88.

3.8.1 Design Specifications

A successful design cannot be accomplished without the appropriate design specifications. Therefore, before any of the established design methods can be used to produce a design, the design specifications have to be clarified. Figure 89 shows the order in which the user is asked for different aspects of the specifications by the expert system.

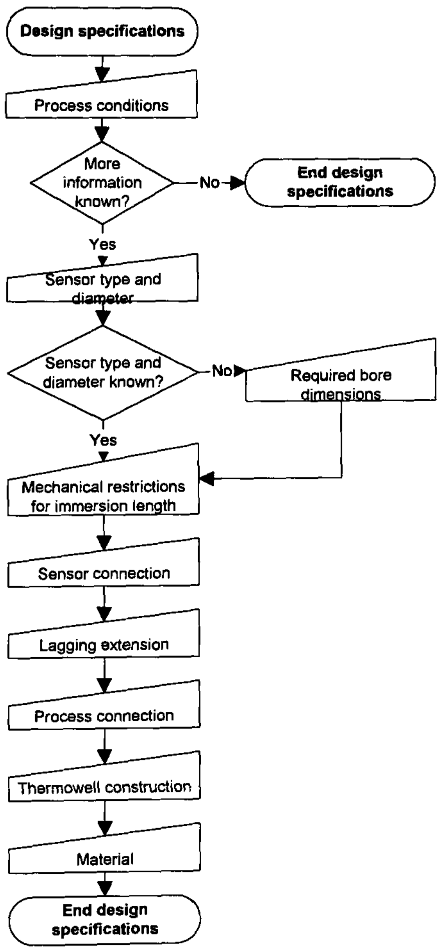


Figure 89: Order of design specification input

First the process conditions have to be identified. These are the flow medium, the flow velocity or flow rate, the operating temperature, the operating pressure and, if the flow rate has been specified, the pipe or vessel diameter. Similar to the thermowell analysis feature, an interface was designed that allows to describe the design specifications, in conjunction with the information concerning the customer and the order. During the development of the procedure concerning the input of the specification information as described in Figure 89 it was decided to display separate windows for the different characteristics. Using one ***Session*** window for all the possible specification points that can be entered would look rather untidy. Therefore, the ***Session*** window only contains the images to specify the customer and order information and the process conditions, which are usually known. Figure 90 shows this input screen.

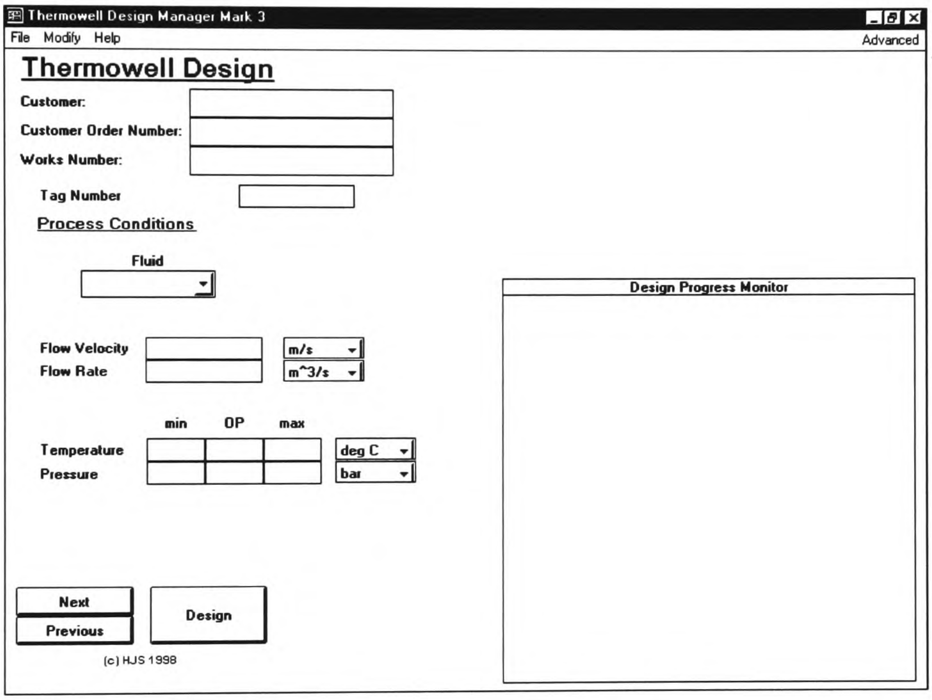


Figure 90: Input screen for thermowell design

A ***TranscriptImage*** was added to the screen. This image will display all the actions that are carried out during the design synthesis process and allows the user to trace each design step. The Button images 'Next' and 'Previous' which are also present in the window have no function in the current version of the expert system. They were intended to provide the user with the same facility to design more than one thermowell at a time as is possible in the thermowell analysis feature.

Generally, all the properties in the input window are known by the client. In the case that one or more of the process conditions are not known, the following assumptions are made by the expert system:

- the flow medium is air
- a maximum flow velocity suitable for the established design will be calculated
- the operating temperature is assumed to be the room temperature, i.e. 20°C
- the operating pressure is at standard atmospheric pressure, i.e. 1.01325 bar

Also, a tag number for the thermowell can be specified at this point. If none is specified, a default tag number is used. (The tag number is required for internal use by the expert system.)

When the 'Design' button is selected, the flow velocity, if specified either directly or as a flow rate, is checked whether it is larger than the recommended maximum velocity of 91.44m/s (Murdock 1959). If this is the case, then the user is notified and can choose whether to stop the design procedure or continue anyway. Then the user will be asked if more information is known about the application (see Figure 89). If this is not the case then the synthesis process will begin. Otherwise, the user answers a series of additional questions concerning the restrictions on the thermowell design. For each question, a separate window with the appropriate ***Text***, ***Edit*** and ***ComboBox*** images is displayed; see Figure 91 for an example. This particular window asks the user to specify any mechanical restrictions for the thermowell, namely the minimum and maximum immersion lengths and the maximum possible outside diameter. For an explanation of these restrictions see page 144 and also section 3.8.6.

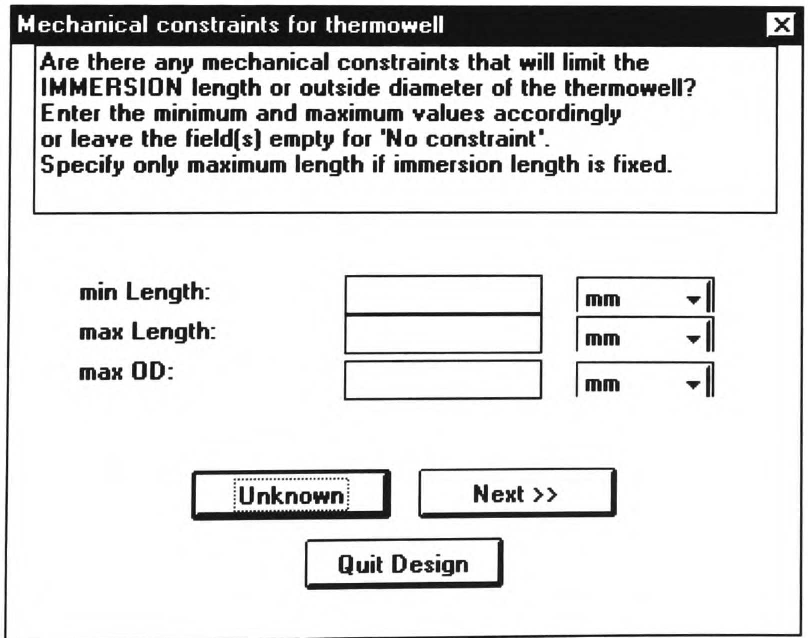


Figure 91: Example for a window asking for additional design specifications

Each window has three ***Button*** images. The first button, 'Unknown', allows the user to indicate that this particular information is not currently known. The button 'Next>>' is used when the information required in this window has been specified. Finally, button 'Quit Design' stops the design procedure.

In the first window the sensor that will be used in the thermowell can be described. The user can select from three sensors: RTD, Thermocouple or Others. Others can be used for bi-metallic or mercury-in-steel thermometers, for example. For all sensor types, the diameter of the stem can be entered. Should RTD or Others have been selected, the user can also enter the length of the sensing area or bulb. It is a requirement for those instruments that the sensing area is completely immersed in the fluid to give accurate temperature measurements.

Alternatively, if there is no information about the sensor available (selection of button 'Unknown'), the user can specify a required bore diameter instead. If this information is also unknown, the expert system uses a bore diameter of 13mm which ensures that most of the available temperature sensors and indicators can be used with the thermowell.

Following this, limitations on the minimum and maximum immersion length of the thermowell can be set, together with the maximum outside diameter possible if such a constraint exists. The length of the thermowell can be restricted by the diameter of the pipe and the length of a connection nozzle fitted with the mating flange, for example. Figure 92 illustrates this point. It can also be seen that the diameter of the nozzle limits the maximum diameter of the thermowell.

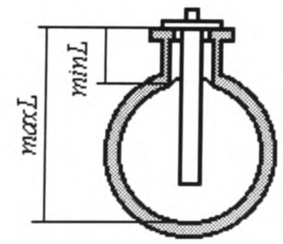


Figure 92: Restrictions for thermowell length

If the length restrictions are not specified, but the length of the sensing area of the sensor used in the thermowell is known, then the expert system sets that length as the minimum length.

Also, should both the length restrictions and the sensitive length be given, the larger of the two lengths is used, thus ensuring complete immersion of the sensor.

If the immersion length of the thermowell is fixed, e.g. because the length of the sensor is fixed, then this can be specified, too.

Then the type and size of the process connection can be specified. This is necessary when the vessel or pipe is already fitted with a connection; the thermowell has to be compatible with that connection. The selection of a process connection size will also restrict the maximum outside diameter of the thermowell. If both process connection and a maximum outside diameter are specified, then the smaller of the two will be used as a limit.

Finally, the user has the option to specify a thermowell construction and the material that is required for thermowell manufacture, provided this information is known.

After the design specifications have been established, the design process will continue with the selection of the material.

3.8.2 Material Selection

At this point the expert system was supposed to choose a material suitable for the given application. But as discussed earlier it was not possible to establish a satisfactory solution for this design step. Therefore, if the client has not specified a material during the design specifications phase 316 stainless steel will be used, as recommended (section 2.2.4).

Also carried out in this design step is the determination of all the necessary material properties at the given process temperature. These properties are the modulus of elasticity E, the specific weight G and the maximum allowable stress S. In situations where the temperature error also needs to be established, the specific heat C_p and thermal conductivity k of the material are also determined. The values are established using the lookup tables in the database as described in section 3.6.5.

3.8.3 Pressure Considerations

Before any other mechanical requirements can be investigated it is necessary to establish the minimum wall thickness, and therefore the outside diameter a thermowell must have in order to withstand the pressure it is exposed to - and from which it has to protect the temperature sensor.

As the tip conditions specify the allowable pressure, the outside diameter at the tip must be determined. It is not only the outside diameter that affects the maximum pressure, however, the bore diameter is also of significance. The minimum value for the bore diameter is fixed by the design specification. The specification will either state the diameter of the sensor that has to fit in the thermowell, or the bore diameter itself is given. In either case, it is not possible to decrease the bore diameter. Therefore the outside diameter of the well has to be increased until the wall thickness is suitable for the operating pressure. The bore diameter, if not directly specified, depends on the temperature sensor specified by the user. If a RTD is used, the bore diameter is set to be 0.25mm larger than the stem diameter of the RTD, otherwise the gap between stem and thermowell wall can be larger. It is therefore set to 3mm larger than the stem diameter of the sensor to allow easy assembly.

The pressure a thermowell can withstand is calculated using the equation $P=K_1S$. The constant K_1 is a function of tip diameter B and bore diameter d. However, the relationship is not straightforward and it is therefore not possible to solve the equation for B, giving the minimum diameter. Instead, a value for B is chosen, K_1 and P are calculated and then P is compared with the operating pressure P_O. If the calculated allowable pressure is larger than the operating pressure, the diameter and therefore the wall thickness is suitable for the application. Otherwise, the diameter should be increased further until the pressure criteria is satisfied.

The way the diameter is changed depends on the type of thermowell construction - solid or fabricated.

3.8.3.1 Solid Construction

In the case of thermowells drilled from solid barstock, the method is straightforward (see Figure 93).

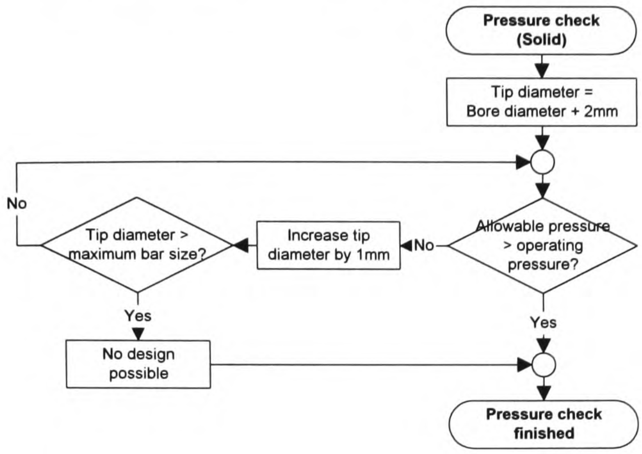


Figure 93: Pressure considerations for solid thermowells

In the first attempt, the value for the tip diameter B is set to be 2mm larger than the bore diameter d, i.e. $B=d+2$mm. All necessary values will now be calculated and if the calculated allowable pressure exceeds the operating pressure, the first design step is finished. Otherwise, the outside diameter is increased further. Each time this operation is executed, the diameter will be increased by 1mm, i.e. $B_n=B_{n-1}+1$mm. However, this increase can only be carried out until the maximum diameter of the material used is reached. In this case it is not possible to design a thermowell that can withstand the pressure; the design procedure is aborted and the user is informed of the result.

3.8.3.2 Fabricated Construction

Fabricated thermowells are manufactured from tubing to avoid machining. This limits the choice of diameters, as the diameters depend on the size of the tube used.

Figure 94 shows the process of determining the pressure criteria for fabricated thermowells.

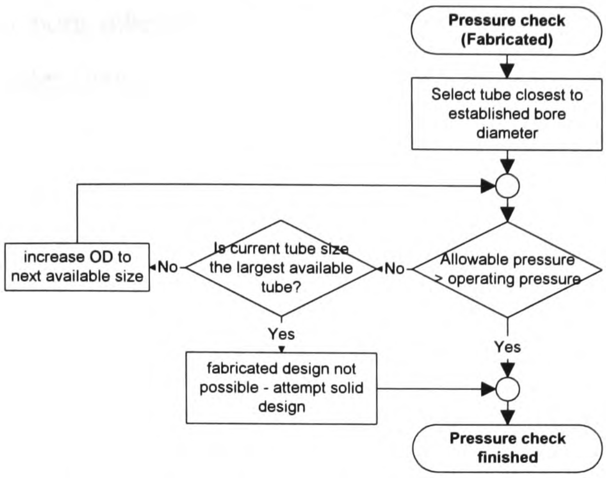


Figure 94: Pressure considerations for fabricated thermowells

First, a tube is selected by matching its inside diameter to the specified bore or sensor diameter. Then, the allowable pressure for that tube is calculated using the familiar equation. Is the operating pressure larger than the allowable pressure, the next larger tube is used and the process is repeated.

However, when increasing the tube size not only the outside diameter changes, but also the tube's wall thickness and therefore the bore diameter. This in turn can decrease the value of K_1, therefore decreasing the maximum allowable pressure. Table 23 compares the values for K_1 for two successive tube sizes. The first tube is selected according to the required bore diameter (either specified directly or established from the sensor diameter). In the example, a bore diameter of 13mm is assumed, resulting in a tube with ½" nominal bore, and a wall thickness of 3.73mm (Schedule 80s). If this tube does not satisfy the requirements (this can be due to failure predicted by the pressure, stress or vibration analysis), the wall thickness of the next tube size has to be selected which is closest to the required bore diameter, resulting in a ¾" Schedule 80s tube.

Table 23: Comparison of K_1 for two tube sizes; tube information courtesy RGB Stainless Ltd

	½" Sch 80s	¾" Sch 80s
Outside diameter [mm]	21.34	26.67
Wall thickness [mm]	3.73	3.91
Bore diameter [mm]	13.88	18.85
K_1	0.207	0.166

Calculating K_1 for both tubes (see Table 23) it can be seen that K_1 for the larger tube size is 20% smaller than K_1 for the smaller tube. Accordingly, this results in a allowable pressure for the ¾" tube that is 20% smaller than the allowable pressure of the ½" tube. As has been established previously, the stress constant K_1 depends on both the outside and bore diameter. Because the wall thickness is increased less than the outside diameter of the tube, resulting in a larger bore diameter, the stress constant K_1 consequently is smaller. The value of K_1 can only be increased by increasing the wall thickness proportionally to the outside diameter. For example, increasing the wall thickness by 25% (the same as the outside diameter increase) will result in K_1=0.207, the same value as for the ½" tube. Keeping the bore diameter at the same value d=13.88mm, hence increasing the wall thickness by 140%, results in K_1=0.335, a 61% increase. However, the dimensions for each tube are fixed and these changes are not possible.

Because of the way the wall thickness changes between different tube sizes it is not guaranteed that a suitable tube can be found. Also, for practical reasons, the largest tube size available is 1" nominal bore. Should it not be possible to find an appropriate tube, the fabricated thermowell design will be aborted and a solid thermowell will be designed instead. If a fabricated thermowell has been specifically requested by the client, they will be asked to decide between carrying out a solid design or aborting the procedure.

3.8.4 Vibration Considerations

Having established the required outside diameter at the tip of the thermowell the relationship between the wake frequency and natural frequency of the thermowell can be determined.

The wake frequency is calculated with the equation $f_w = 0.22\,{}^{v}/_{B}$; the natural frequency is determined using the appropriate approach for either parallel, reduced parallel or tapered thermowells. It is important to note that not all the relevant information has necessarily been specified by the user. It may therefore be necessary for the expert system to make some assumptions about the immersion length or the flow velocity.

After the natural frequency has been calculated, the frequency ratio r is established. If r is satisfactory, i.e. $r \leq 0.8$, the next design step (stress considerations) can begin. Otherwise the design has to be modified and the new natural frequency, possibly the wake frequency and the frequency ratio may need to be re-calculated. This process continues until a suitable design has been established or it can be assumed that no design is possible for the given specifications.

The way the calculations are carried out and modifications are implemented depends on the information given by the user (see Figure 95):

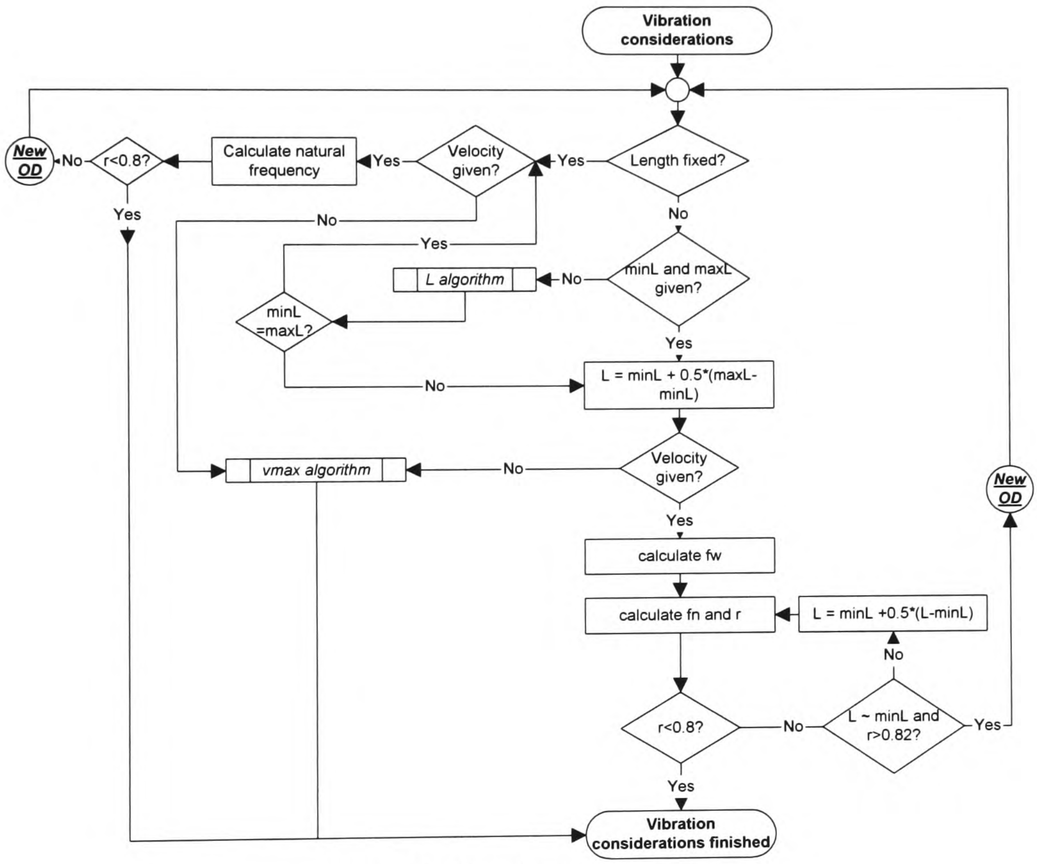


Figure 95: Vibration considerations

The various possible combinations resulting from this structure are described in detail in the chapters indicated in Table 24.

Table 24: Conditions and relevant chapters for the vibration considerations

Condition 1	Condition 2	Condition 3	Refer to section
Immersion length for thermowell fixed	flow velocity/rate known	solid construction	3.8.4.1.1.1
-"-	-"-	fabricated construction	3.8.4.1.1.2
-"-	flow velocity/rate not known	N/A	3.8.4.1.2
Minimum and maximum immersion length specified	flow velocity/rate not known	N/A	3.8.4.2.1, 3.8.4.1.2
-"-	flow velocity/rate known	N/A	3.8.4.2.2
No immersion length specified	flow velocity/rate known	N/A	3.8.4.3.1
-"-	flow velocity/rate not known	length of sensing area known	3.8.4.3.2.1
-"-	-"-	length of sensing area not known	3.8.4.3.2.2

3.8.4.1 Immersion length fixed

3.8.4.1.1 The flow velocity or flow rate is known

3.8.4.1.1.1 Solid construction

The initial attempt is carried out using a parallel design (see section 2.2.3). Should this design not prove to be acceptable, then the root diameter, the tip diameter or both could be increased. However, in the parametric study it was shown that increasing only the root diameter will give a higher natural frequency than increasing both root and tip diameter (see section 2.2.1). Therefore, only the root diameter is increased at this stage if either no sensor has been specified or a thermocouple is used. Figure 96 shows the procedure used when the outside diameter has to be increased, depending on the type of thermowell and the information known about the sensor used in the application.

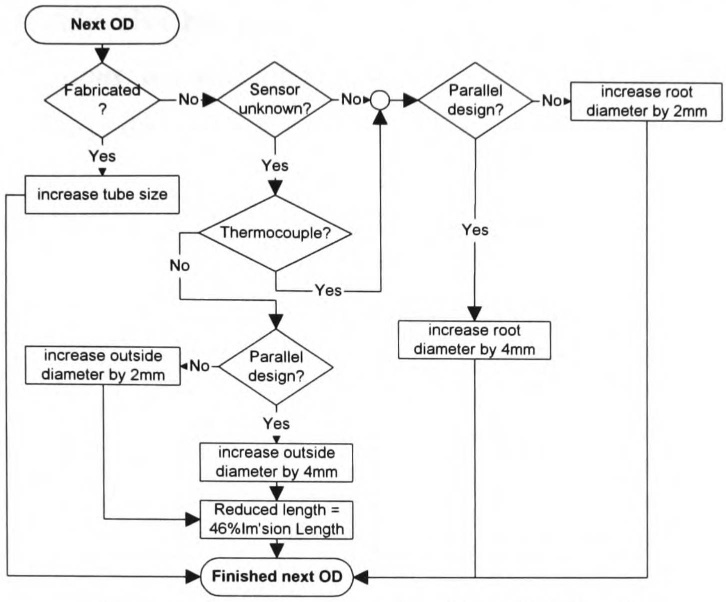


Figure 96: Increase of thermowell outside diameter

The increase will be 4mm initially, and 2mm in each subsequent attempt. In cases where the sensor is specified as either RTD or Other, a reduced parallel design is attempted. The increase of the outside diameter is carried out in the same manner as described for the tapered thermowell. The reduced length is set to 46% of the immersion length. This value was chosen to keep the length shorter than half the overall length and therefore achieving a high natural frequency (see section 2.2.1.2.5). After each calculation and check of r the current root diameter is compared with the diameter for the process connection or the maximum possible diameter, if either of them has been specified by the user. This is necessary to avoid designing a thermowell which will not be compatible with the pipe or vessel. Should the new root diameter be larger than the restricting diameter, the tip diameter will be increased instead. Then, the design process starts again with a parallel geometry. If the new tip diameter is also larger than the specified process connection diameter or maximum possible diameter, then the design process is aborted.

3.8.4.1.1.2 Fabricated construction

The tube size will be increased until the frequency ratio is satisfactory or the outside diameter of the tube conflicts with the specified process connection or maximum possible diameter. In the latter case a solid design will be attempted, see section 3.8.4.1.1.1 *Solid construction*. It is also important to check the pressure criterion once the tube size has been increased. As discussed in the section 3.8.3 *Pressure Considerations,* the wall thickness changes when the tube size is changed. Therefore, it is possible that a larger tube will fail the pressure criterion, resulting in a switch to solid design.

3.8.4.1.2 The flow velocity is not known

In this case the following procedure is applied to establish the maximum flow velocity possible for the specified immersion length (see Figure 97).

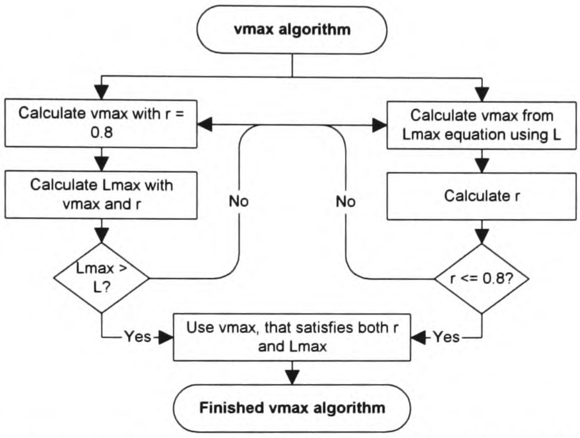


Figure 97: Determination of suitable immersion length and flow velocity

It was established during the review of the design procedures that whether a thermowell fails the vibration or the stress analysis first depends not only on the flow velocity and thermowell length, but also on the pressure and the density of the process fluid. Therefore, it is necessary to consider both critical cases in order to find the maximum possible velocity for a given thermowell length. Looking at the frequency ratio, the maximum velocity possible will be for the case of r=0.8. The maximum velocity possible in terms of the maximum allowable length is given when $L=L_{\max}$.

First, the critical case of r=0.8 will be discussed. The maximum velocity at this frequency ratio can be calculated from the standard equation

$$r = \frac{f_w}{f_n}$$

with $\quad f_w = 0.22{}^{v}/_{B}$ and $r = 0.8$,

thus, $\quad v_{\max} = 3.636 f_n B$

The natural frequency is calculated according to the equation for parallel, reduced parallel or tapered thermowells.

Using r and the calculated value for v_{max} the maximum allowable length for the thermowell can now be established and compared with the fixed immersion length. If the immersion length is shorter than the allowable length then the velocity determined above is the maximum allowable velocity.

Should this not be the case then the second critical limit is of importance. Using the equation for the maximum allowable length

$$L_{max} = \frac{K_2}{v}\sqrt{\frac{\upsilon(S - K_3 P_O)}{1 + F_M}}$$

and setting L_{max} equal to the specified immersion length, the maximum velocity can be calculated

$$v_{max} = \sqrt{\frac{\xi}{1 + \xi\chi^2}}$$

with $\xi = \left(\frac{K_2}{L}\right)^2 (S - K_3 P_O)\upsilon$ and $\chi = {}^{0.22}\!/_{Bf_n}$.

Once this velocity has been established, the wake frequency and therefore the frequency ratio r can be determined.

It is now possible for both cases to check the vibration and stress criteria. The velocity for the case where both criteria are satisfactory is the maximum allowable velocity. It will never be the case that neither attempt will be successful, therefore it is always possible to determine v_{max}.

The established maximum velocity is passed on to the customer so that the process can be set up accordingly, thus avoiding a velocity larger than v_{max}.

3.8.4.2 Minimum and maximum length specified

In most cases the restrictions on the immersion length will be caused by the pipe or vessel diameter (see section 3.8.1 *Design Specifications*). It can be assumed that the temperature of interest is in the middle of the pipe or vessel. This assumption is based on the common temperature profile in a pipe, which is of a similar shape to a velocity profile with the maximum temperature in the middle of the pipe and decreasing temperature towards the pipe walls. Therefore, the immersion length will be set initially to that length, i.e. L_0=*minL*+0.5(*maxL*-*minL*). If the solid thermowell construction is required, the first design attempts are carried out using a parallel thermowell.

3.8.4.2.1 The flow velocity is not known

If the flow velocity is not known at this stage, then the same procedure as discussed for a thermowell with fixed immersion length is applied, see section 3.8.4.1.2 and Figure 97. In this case the fixed immersion length is L_0, as determined above.

3.8.4.2.2 The flow velocity is known

If the flow velocity is known then the natural frequency and the frequency ratio can be calculated. It has been established in the parametric study and in the practical vibration analysis that the natural frequency of any thermowell increases with decreasing immersion length (see chapter *Discussion of Design Methods - Vibration Analysis*). Therefore, if the vibration criterion is not satisfactory the immersion length will be decreased. This new length is set to a length halfway between the minimum length and the current immersion length, i.e. L_1=*minL*+0.5(L_0-*minL*). Because a minimum and maximum length are specified it can be assumed that the customer will accept thermowells within those limits, therefore it is possible to decrease the length. The immersion length is reduced until r<0.8, or the minimum length has been reached. If the vibration criterion is satisfied the design process will continue with the stress considerations. Otherwise, the outside diameter of the thermowell has to be increased, which depends on the current construction and geometry of the thermowell. For fabricated thermowells, the tube size will be increased, unless no larger size is available. This will initiate a solid design attempt. In the case of solid thermowells, the root diameter will be increased. See also Figure 96 on page 152 and section 3.8.4.1 for a discussion of the different aspects that have to be considered when changing the outside diameter.

After the tube change or diameter increase, the new diameter will be compared with the specified process connection or limiting diameter. Should the new diameter conflict with the specifications, the following cases can occur:

- If the thermowell is fabricated, solid design will be attempted
- If the thermowell is solid and parallel then the design must be aborted.
- If the thermowell is tapered or reduced parallel, the tip diameter will be increased and parallel design is attempted again.

In either case (unless the design is aborted) the pressure criterion is applied first and the immersion length is set to L_0 (see above) for the vibration considerations.

3.8.4.3 No immersion length specified

If there are no specifications given about the possible range for the immersion length of the thermowell then some assumptions are made to decide on the immersion length to be used. Immersion lengths of only a few millimetres are unlikely to give good measurement accuracy even though they will withstand very high flow velocities; thermowells of several metres will have a short fatigue life unless the flow velocity is virtually zero. Therefore, the decision on the immersion length was based on the various specifications that influence the length, such as flow velocity and length requirements given by the temperature sensor. Again, this depends on which of the appropriate information is given (Figure 98).

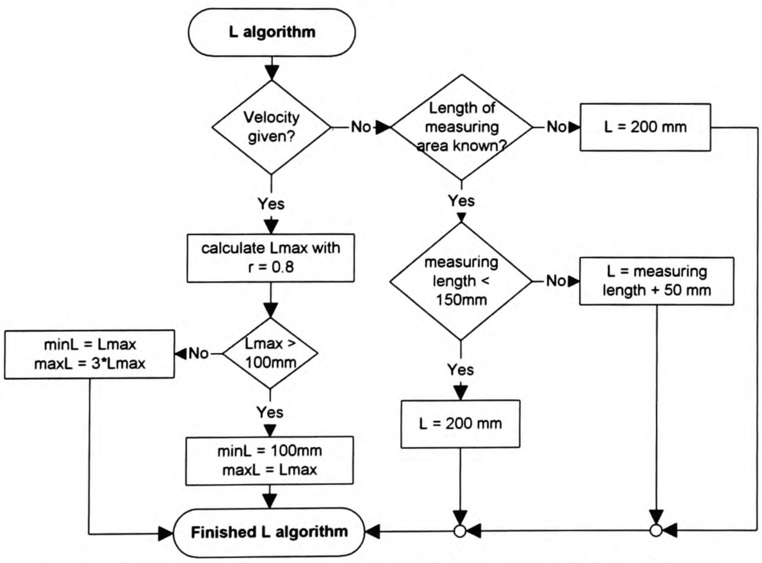


Figure 98: Procedure for establishing an immersion length

3.8.4.3.1 The flow velocity or flow rate is specified

In this case the maximum allowable length for the thermowell is calculated (see section 3.8.5) using a frequency ratio of r=0.8. If this length is larger than 1000mm, then the maximum length will be set to *maxL* = 1000mm and the minimum length is set to *minL* = 100mm. 1000mm was chosen as a maximum value for the length because at this length the natural frequency of the thermowell is very low (see section 2.2.1.2.5), making such a thermowell suitable only for applications with small flow velocities. The minimum value of 100mm, however, can be used at much higher flow rates.

Should the calculated maximum allowable length be smaller than 100mm, the minimum length will be set to that value and the maximum length is set to 3 times the maximum allowable length. In any other case, the minimum length is set to 100mm and the maximum length to the calculated L_{max}.

3.8.4.3.2 The flow velocity or flow rate is not specified

3.8.4.3.2.1 The length of the measuring area is known

If the length of the measuring area is larger than 150mm, then the immersion length is set to the length of the measuring area + 50mm, i.e. $L = L_{MeasuringArea} + 50mm$; otherwise, the immersion length is set to L=200mm. The additional 50mm increases the likelihood of the sensor being immersed completely in the fluid if the customer was not aware of any minimum length requirements. The recommendation of using a thermowell which is three times longer than the sensing area of the temperature sensor (see section 2.1.3) is not implemented at this stage. This increases the chances of producing a suitable design. However, at the end of the design process the immersion length is compared with the length of the sensitive section and the user is informed whether it conforms with the above recommendation. At this stage the client might request a larger immersion length.

3.8.4.3.2.2 The length of the measuring area are not specified

In this case, the immersion length is set to 200mm. This immersion length offers a high natural frequency, whilst making sure that most stem sensitive instruments are completely immersed in the fluid. The sensitive section of the majority of sensors has a length of 50mm or less. Including Jones' recommendation (see section 2.1.3) results in an immersion length of 150mm. An additional 50mm were added to include the possible use of a nozzle.

After all these assumptions have been made the design will now be dealt with as if either a minimum and maximum length (first case, flow velocity or flow rate is known) or a fixed immersion length (second and third case) have been specified by the user.

It has to be noted at this point, that even though the consideration of the process connection and the maximum possible diameter are mentioned in the cases of 'immersion length fixed' and 'maximum and minimum length specified', they are not considered as part of the *Vibration Considerations* design step in the finished expert system. In fact, even if the vibration criteria is not successful, the expert system will continue with the next design step, *Stress Considerations*. Only after all steps have been considered will an evaluation take place to establish what changes have to be made to achieve a satisfactory design. However, it was necessary at this stage to indicate all aspects that have to be considered in the design process, and how they affect the decisions that are made throughout.

3.8.5 Stress Considerations

The combined stresses caused by the operating pressure inside the vessel and the bending force due to the flow around the thermowell pose a limitation on the thermowell length. To determine this length equation [7] is used.

If the calculated length L_{max} is larger than the immersion length, i.e. $L_{max} > L$, then this step is finished and the process connections can be considered. Otherwise, changes have to be made to either decrease the immersion length or increase the maximum allowable length. Again, the changes possible depend on the design specifications (see Figure 99):

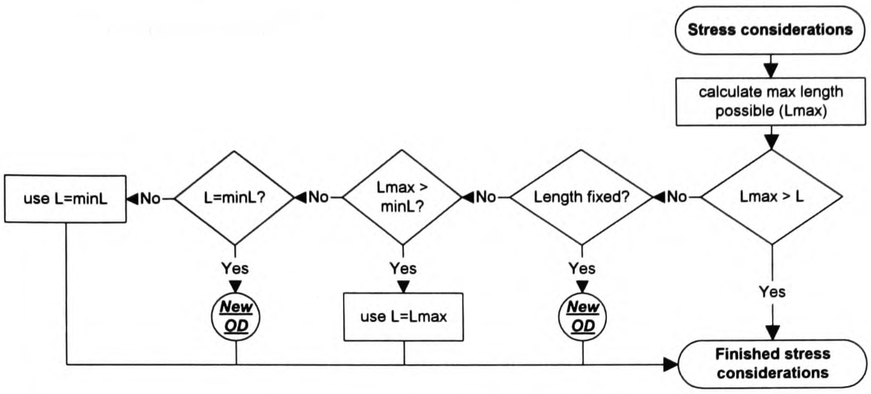


Figure 99: Stress considerations

The different options are explained in more detail in the sections as indicated in Table 25.

Table 25: Conditions and relevant chapters for the stress considerations

Condition 1	Condition 2	see section
immersion length fixed	N/A	3.8.5.1
minimum and maximum immersion length specified	L_{max} > minL	3.8.5.2.1
-"-	L_{max} < minL	3.8.5.2.2
no immersion length specified	N/A	3.8.5.3

3.8.5.1 *Immersion length fixed*

In this case the length of the thermowell cannot be decreased; hence, measures have to be taken to decrease the stresses at the root and therefore increase the maximum allowable length. Increasing the outside diameter will decrease the stresses at the fixed end. This change of the outside diameter is carried out in the same way as for the vibration criteria (see section 3.8.4.1).

3.8.5.2 *Minimum and maximum length specified*

There are several possible instances, as the calculated allowable length can not only be smaller than the current immersion length ($L_{max} < L$), it could also be smaller than the specified minimum length ($L_{max} < minL$).

3.8.5.2.1 L_{max} > minL

A comparison between the calculated maximum length L_{max} and the specified minimum immersion length *minL* is carried out. If the maximum length is larger than the minimum immersion length (L_{max} > *minL*) then the current immersion length is set to L=L_{max}.

When re-calculating the stress criterion the immersion length is now shorter than the newly calculated maximum allowable length. Therefore, the stress criterion has been satisfied and the current design step is finished.

3.8.5.2.2 L_{max} < minL

In the case that the maximum allowable length calculated is smaller than the minimum length, i.e. L_{max}< *minL*, the immersion length is set to the minimum length specified, L=*minL*. This will not necessarily produce a satisfactory design in respect of the stress criterion. It is possible that, when the maximum allowable length is re-calculated, L_{max} is still smaller than *minL*. As the immersion length cannot be reduced any further, the thermowell diameter has to be increased to reduce the stresses at the fixed end. This is carried out the same way as for a thermowell with fixed immersion length.

3.8.5.3 No immersion length has been specified

At this point the case of a thermowell with no specified immersion length does not exist any more even if no restrictions were specified by the user. This is due to the fact that whilst considering the vibration criterion, limits were set on the immersion length - either by designating a fixed length or minimum and maximum values, depending on the circumstances (see section 3.8.4.3).

3.8.6 Process Connection Considerations

This phase of the design process either establishes what type and size of process connection is best used for the application, or it checks whether the current design is compatible with the process connection specified by the user. Also considered in this section is the maximum outside diameter, if specified by the user. This dimension represents a limitation for the outside diameter of the thermowell. One reason for specifying a maximum diameter could be the use of a nozzle on the vessel, Figure 100.

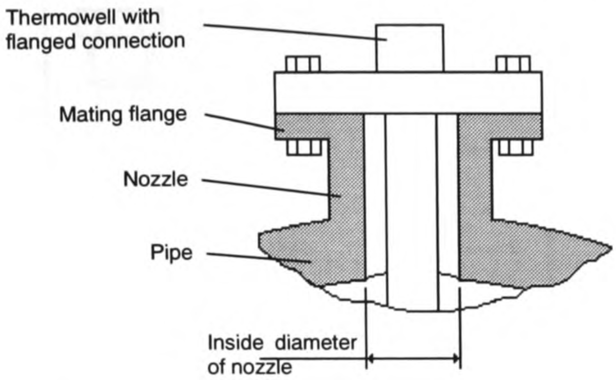


Figure 100: Example for maximum diameter

From this example it is clear that the inside diameter of the nozzle poses a restriction for the outside diameter of the thermowell. The restricting dimension for this example is either the limiting diameter of the flange (see later on in this section) or the nozzle inside diameter, depending on which one is smaller.

The generalised procedure for the process connection design step is shown in Figure 101.

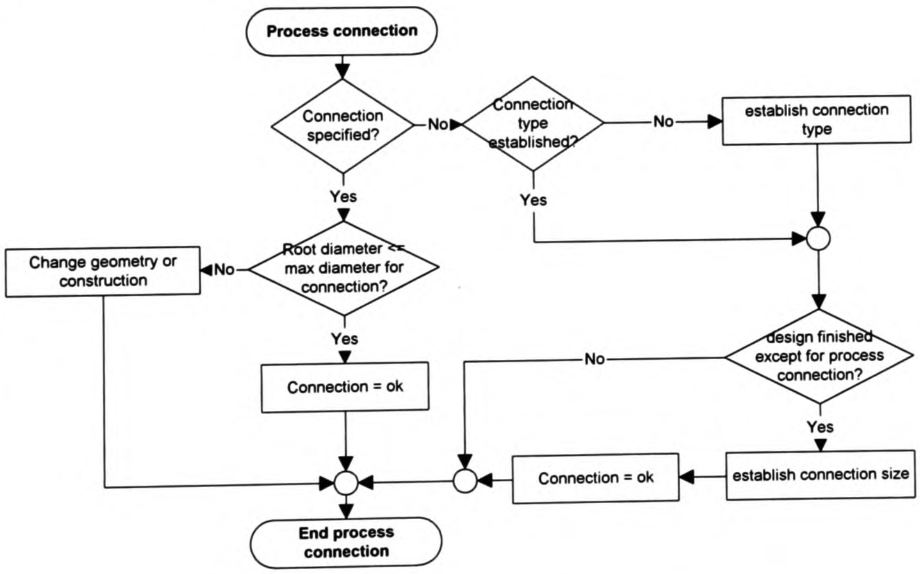


Figure 101: Process connection considerations

If no attention is paid to the process connection it is possible that the thermowell cannot be fitted to the process plant or pipe; see Figure 102 for examples using a threaded process connection.

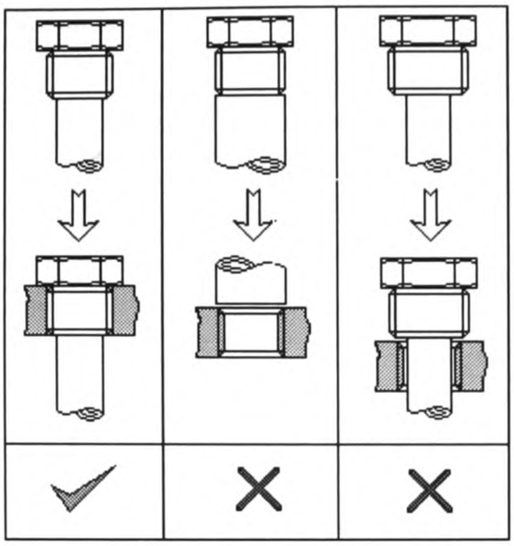


Figure 102: Correct and incorrect process connection

Table 26 indicates the relevant section for the different cases that can be encountered.

Table 26: Relevant sections for the process connection considerations

Condition 1	Condition 2	see section
A process connection is not specified	N/A	3.8.6.1
A process connection is specified	solid construction	3.8.6.2.1
-"-	fabricated construction	3.8.6.2.2
A limiting diameter is specified	N/A	3.8.6.3

3.8.6.1 Process connection is not specified

For this case a two step approach is used. First, the type of process connection (thread, flange or weld-in socket) suitable for the application has to be selected. Then, once the correct geometry for the thermowell has been determined, the actual size of the connection is established. This task is carried out in two steps to avoid a constant adjustment of the connection size when the root diameter has to be changed in order to adapt the thermowell to the process conditions (see the sections on *Vibration Considerations* and *Stress Considerations*). Figure 103 outlines the process.

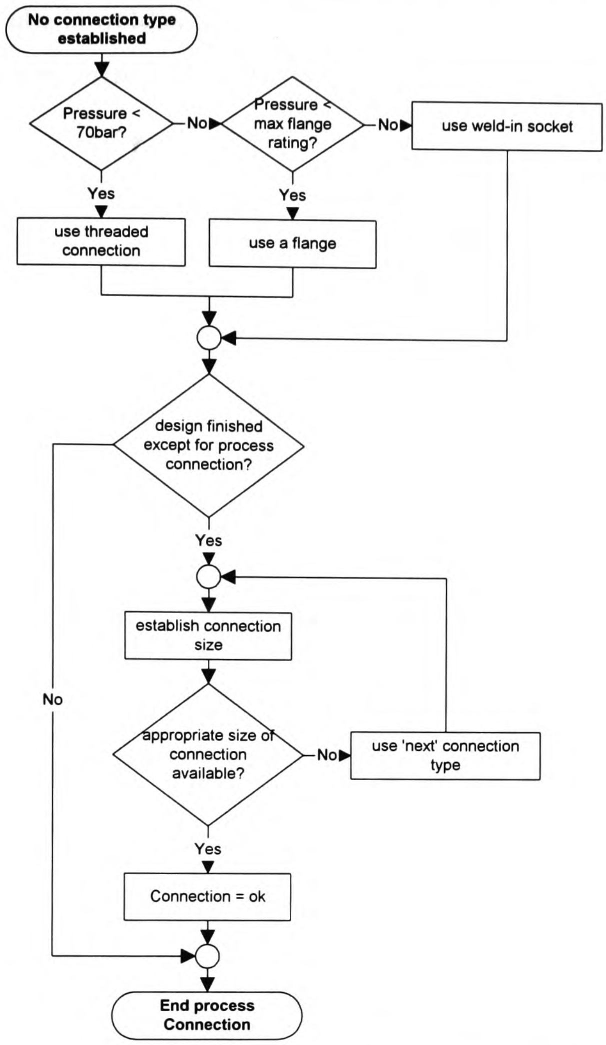


Figure 103: Process connection is not specified

The type of process connection depends on the operating pressure present in the process. The expert system will choose a threaded connection for pressures of 70 bar or less, a flanged connection for pressures higher than 70 bar and a weld-in socket connection for pressures that exceed the maximum flange rating of 2500lb, i.e. 172 bar (see section 2.2.5 *Process Connection Consideration*).

If the thermowell design currently proposed does not fulfil all necessary requirements, modifications will be made accordingly until the design is satisfactory. Only then will the size of the process connection be established.

The size of the connection is established according to the root diameter of the final thermowell design and the limiting diameter for each connection size. For threaded connections the limiting diameter is the gauge diameter. However, a diameter smaller than the gauge diameter is used to avoid problems with fabricated thermowells, as the tube is welded to a separately machined connector. In the case of flanges the raised-face diameter - 20mm is used to allow for the necessary weldments. The limiting diameter for weld-in sockets is the diameter of the material used for their manufacture.

To find the appropriate connection size, the expert system compares each connection size, starting with the smallest, with the root diameter until it finds the first one that is equal to or larger than the root diameter. This connection size will then be used. If a weld-in socket has to be used, the diameter of the socket will be set 6mm larger than the root diameter of the thermowell. Should the largest size of one connection type not be large enough, a different type of connection is used and the size of the new connection is established in the same way as shown before. It is important, however, that the new connection type is suitable for the pressure, therefore the change will only occur in the order thread - flange - weld-in socket.

If it is not possible to determine a suitable connection the design procedure is aborted and the user is notified of this fact.

3.8.6.2 Process connection is specified

If the process connection has been specified by the user, a comparison between the current root diameter and the limiting diameter of the connection is carried out every time changes are made to the thermowell geometry, regardless whether the design is satisfactory. This way, conflicts between the two dimensions are noticed and the necessary changes can be made at an early stage, thus cutting down on processing time. Figure 104 shows the procedure when the connection has been specified.

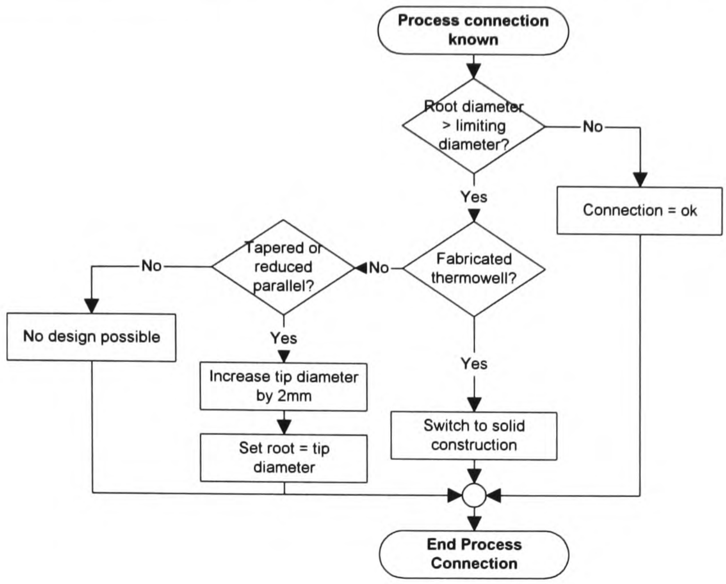


Figure 104: Process connection is specified

If the root diameter is smaller than the limiting diameter, the connection is suitable for the application at this stage and this design step can be finished. Otherwise, changes that depend on the thermowell construction have to be made.

3.8.6.2.1 Solid thermowells

If the current design is either tapered or reduced parallel, then the tip diameter will be increased by 1mm, and the root diameter is set to the same value as the tip diameter, thus creating a parallel design. The pressure, stress and vibration criteria have to be re-calculated for the new thermowell. Further changes to the design will be made if necessary, for example reducing the length or increasing the root diameter, until the design is satisfactory or it is evident that a suitable design cannot be established for the given application.

If the current thermowell design is parallel then the tip diameter cannot be increased further, as it still will be larger than the limiting diameter. Therefore, a thermowell with the given specifications cannot be designed. In this case the process is aborted and the user is informed.

3.8.6.2.2 Fabricated thermowells

The tube currently used for the thermowell has been chosen to satisfy the pressure, stress and vibration criteria. In fact, it might be necessary to increase the tube size further should one (or more) of the criteria not be satisfactory. It is therefore not possible to reduce the diameter of the thermowell to suit the process connection. In this case, the expert system will attempt the design of a solid thermowell for the given application.

3.8.6.3 Maximum outside diameter is specified

This case is dealt with in the same way as when the process connection has been specified (section 3.8.6.2), only that the maximum outside diameter is used as the limiting diameter. However, this condition will only be considered if either the process connection has not been specified or the specified maximum outside diameter is smaller than the limiting diameter of the specified process connection.

3.8.7 Thermal Considerations and Estimation of Thermowell Price

Once a design has been established, the user can select whether the temperature error has to be established. The section 3.6.6 explains the additional input necessary and the procedure of this function.

The estimation of the thermowell price is carried out for every thermowell that has been designed. Additional information as discussed in section 3.6.7 does not have to be entered at this stage, because the user has already specified the necessary information, for example the lagging extension, during the design specifications phase. The only additional information that might have to be provided by the user is the price of the used material, in case it is not available in the expert system's database.

3.8.8 Presentation of Results

Once the design has been established, it has to be made available to the user. A new ***Session*** window was created for this purpose, which displays all relevant information concerning the design. It was decided to also include a drawing of the designed thermowell, together with the established or specified process connection. The drawing is not a proper engineering drawing and it is not to scale. However, it indicates the different thermowell characteristics. Figure 105 shows an example for the design details window.

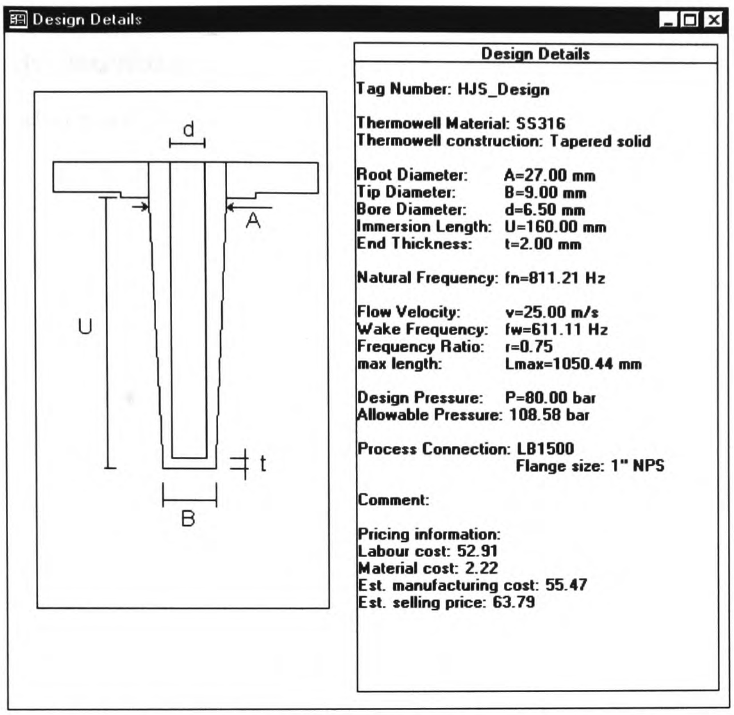


Figure 105: Details of established thermowell design

The window not only contains the details for the thermowell geometry, but also the values for the thermowells natural frequency, the wake frequency, etc. This way the results of the 'wake frequency analysis' can be provided without carrying out the analysis separately. Also specified are the process connection and the pricing information for the thermowell. The 'Comment' section in the design details window is used to display messages concerning decisions made by the user during the design process. For example, if fabricated design has been originally specified but during the design synthesis it was evident that such a design will not be successful, the user can decide whether to stop the process or continue with solid design. If the procedure is continued with a solid construction, then an appropriate message is displayed under 'Comment'.

As in the thermowell analysis, the results can be saved to disk or printed out.

3.9 Features of the Thermowell Design Manager

The features of the expert system dealing with the design analysis and synthesis have been already described in the appropriate sections but a short description of other features is also required.

3.9.1 Main Menu

When the expert system is started, the window in Figure 106 is displayed.

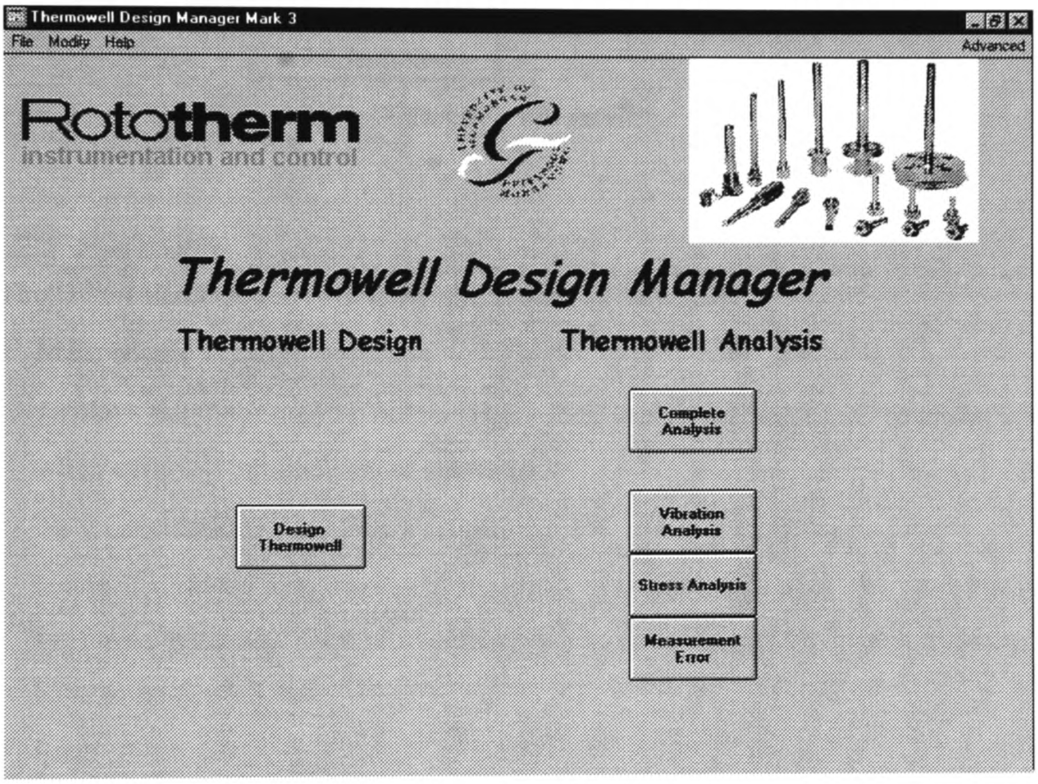


Figure 106: Main menu

This window is used as a main menu from which the user can select whether to analyse an existing thermowell or design a new one. The user also has the choice between carrying out a full analysis, or only the vibration, stress and pressure analysis or to establish the measurement error of the thermowell. Depending on the choice, the appropriate window is displayed (see sections 3.7 *Thermowell Analysis* and 3.8 *Thermowell Synthesis*).

3.9.2 Menu Bar of the Main Menu

The menu bar of the window offers several functions. These functions are also accessible from the thermowell analysis and synthesis windows.

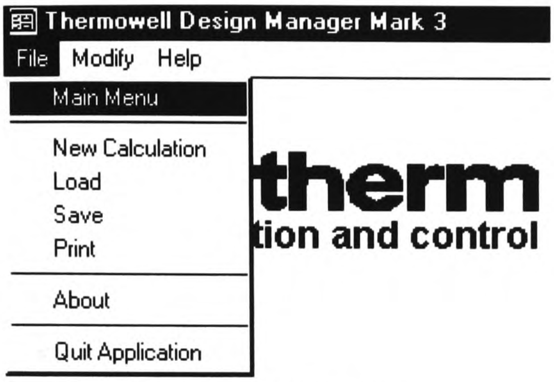


Figure 107: File menu

3.9.2.1 File menu

The 'File' menu (Figure 107) offers the following functions:

- Main Menu

  displays the main menu window (Figure 106). It is therefore only of practical use when carrying out analysis or synthesis.

- New Calculation

  clears all values that have already been specified, either in the analysis or synthesis window. This is necessary when calculations for a new customer have to be carried out.

- Load

  allows the user to load previously saved calculations.

- Save

  enables the user to save the current calculations. This function is only available after an analysis or synthesis has been carried out.

- Print

  sends the results to the local printer.

- About

  displays copyright information and current version of the expert system.

- Quit Application

  closes the application.

3.9.2.2 *Modify menu*

The two functions available in the 'Modify' menu (Figure 108) are used to modify the thermowell specifications and process conditions in the thermowell analysis feature in case a mistake was made during the data input. Choosing the 'Select Thermowell' option displays a window containing a ***ComboBox*** image which displays the tag numbers of the available thermowells. If a thermowell has been selected, the standard data input window for the thermowell analysis is displayed, together with the previously entered specifications. The button used to carry out the calculations is now used to confirm that modifications have been made; its name has therefore been changed to 'Modify'. The option 'Display Modified Results' will display the window containing the analysis results for the modified specifications.

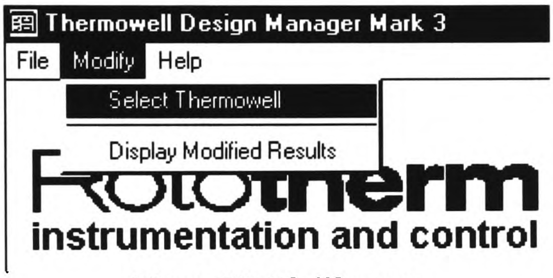


Figure 108: Modify menu

These functions are also available for the design synthesis of a thermowell. As it is not possible to design several thermowells at a time, the modifications are restricted to the current design. Similar to the modification of analysed thermowells, the previously entered specifications are displayed when choosing this option and can be changed by the user.

3.9.2.3 *Help menu*

The Help menu of the expert system is currently not implemented.

3.9.2.4 *Advanced menu*

Using the 'Enter' option of the menu 'Advanced' (Figure 109) the user can enter the functions that deal with the expert system's database. These functions are restricted to certain users and can only be accessed by using a password. The option 'Exit' can only be used once the restricted area has been entered. Similarly, the option 'Enter' is only available from the main menu or any of the design windows. When the user selects option 'Enter', a message is displayed that any data input that might have been made in the design windows will be lost. The user can then choose whether to proceed.

Figure 109: Advanced menu

3.9.3 Restricted Area

The window used in the restricted area is shown in Figure 110.

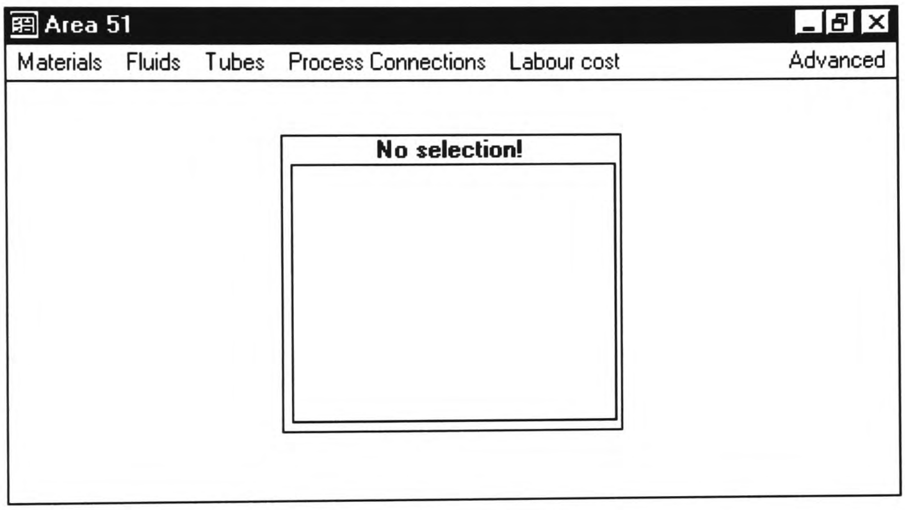


Figure 110: Restricted area

The only image in the window is used to display the available materials, fluids, process connections and tubes. These items are accessible through the different menus in the menu bar.

The 'Materials' menu in Figure 111 is representative for the 'Fluids', 'Tubes' and 'Process Connections' menus. The following options are accessible through those menus and available for the other three menus, too.

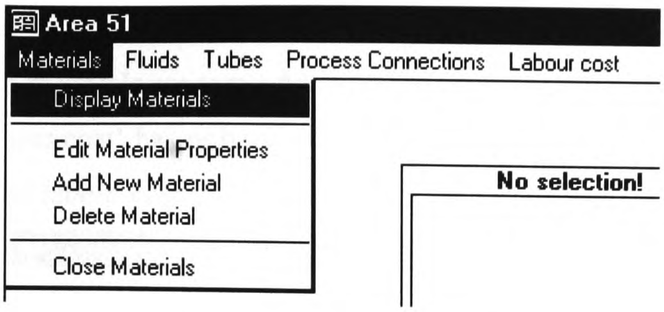


Figure 111: Materials menu

- Display Materials

  displays all available materials.

- Edit Material Properties

  allows to change the properties of materials. The user has to selected the materials whose properties have to be changed first. Figure 112 shows the additional windows that are used to edit the material properties.

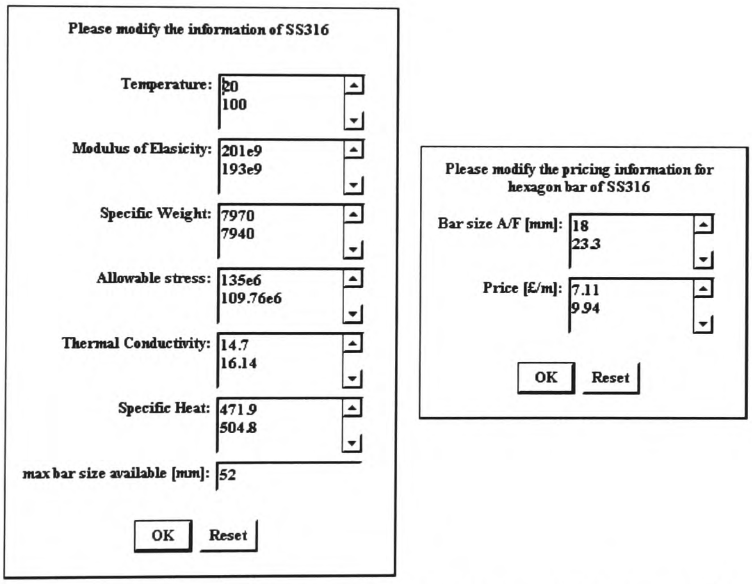


Figure 112: Windows used to enter and edit material properties

- Add New Material

  enables the user to add a new material to the database. The same windows as shown in Figure 112 are used to specify the characteristics of the new material.

- Delete Material

  provides a function to delete materials specified by the user.

- Close Materials

  the displayed materials are removed from the screen.

The menu 'Labour cost' has only one option, Figure 113, which allows the user to change the specified labour cost to a new value.

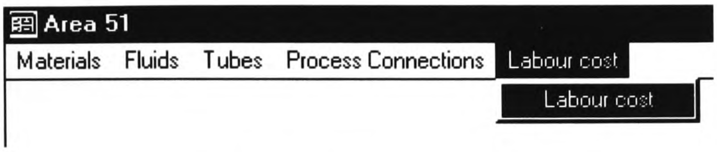


Figure 113: Labour cost menu

Because this area is not accessible to every user, there are no features dealing with unit conversion etc. The values have to be entered in the correct form.

When the user chooses to leave this area by selecting option 'Exit' from the 'Advanced' menu (see Figure 109) and changes have been made to any of the values in the database, the user is asked whether to save the changes or discard them before proceeding.

3.10 Testing of the Expert System

It is important to verify that the expert system produces correct results. During the development of the application, individual functions already had to be tested for their functionality and correct syntax; at the same time, they were also checked whether the expected output is achieved. The results of individual functions for the calculation of maximum deflection, natural frequency, wake frequency etc. can be checked by using test-examples for which the results are already know. These examples are taken from Murdock (1959), ASME (1974), Gibson (1995) and the parametric study.

Once it has been proven that the functions that carry out the calculations dealing with thermowell analysis produce the correct results, the control functions of the application have to be tested. These functions are responsible for the functionality of the application in general. They are used to call the appropriate ***Session*** windows, check the values entered by the user, display results, call the functions used to carry out the calculations, etc.

Testing the functions used by the thermowell analysis feature of the expert system is straightforward, as it was possible to use a sequential, step-by-step programming approach in this case. Syntax and logic errors can therefore be determined quickly. The functions that have to be checked in the thermowell analysis feature are:

- unit conversion

  The value and unit of one of the properties is specified. Then, when the calculations have been carried out, the slot values in class **Order**, which contains the converted values, are compared with manually converted values.

- correct data input

  The user is restricted to use only numbers when entering data, except in the case of tag-numbers, which require both numbers and letters. If a character has been entered that is not allowed, a message will be displayed indicating that the value is invalid and that it has to be re-entered. This is repeated until all values are correct. This feature is simply tested by entering invalid values.

- missing information

  Similarly to the check if valid data has been entered, this feature will display a message to indicate that some of the required information is missing.

- is selected type of analysis the analysis that is carried out?

  The user can select whether a complete analysis or only vibration, pressure and stress or thermal analysis is carried out. The data input window displays a title according to the selection, and after the analysis is carried out only the results that were asked for are displayed. This can be checked by selecting a type of analysis and verifying the title of the input window and the displayed results.

- is the right equation used to determine the maximum deflection?

  Three different equations are used for the three types of thermowells. The calculated maximum deflection is stored in an appropriate slot in the thermowell's instance. This value can be compared with the result achieved with a calculation using MATHCAD. Instead of comparing the value for the deflection, the natural frequency can be used, too, because the same equation is used to calculate the frequency from the deflection for all three thermowell types.

- is the flow velocity calculated when a flow rate has been specified?

  If the user has specified a flow rate and the pipe diameter, the expert system calculates the flow velocity, which is required for the calculation of the wake frequency, using those two values and the fluid's density, if necessary. Flow rate and flow velocity are stored in two different slots, therefore this feature can be tested by checking the slot for the flow velocity.

The thermowell synthesis feature of the application has the same functions as described above, and can be checked in the same way. However, the functions and rules required to carry out the appropriate changes to the thermowell design in order to adapt it to the given application have to be tested, too. To simplify the testing, a ***TranscriptImage*** was added to the ***Session*** window for the input of the design specifications (Figure 90). Each function and rule that is used for the thermowell synthesis displays a message in this image containing information about the criterion currently being checked and the appropriate result, or what changes are made to the thermowell and why. For instance, the rule MinAndMaxLength displays the following message:

Minimum and maximum length specified...
Minimum immersion length : 200 mm
Maximum immersion length : 400 mm
>>Using immersion length of 300 mm...

This rule should only be fired when the conditions, i.e. the IF-part of the rule, are true. In this particular case, the minimum and maximum lengths have to be specified, and an immersion length has not yet been set (see section *Thermowell Synthesis*). If the conditions are true, then the immersion length is set to a value halfway between the two specified extremes. It was decided not to remove this feature, as it allows the user to follow the design procedure.

During the development of the thermowell synthesis feature, a command line was added to the rules and functions that would display the rule's or function's name in the ***TranscriptImage*** whenever it was called. This made it easier to identify each function and rule. These command lines were removed again after the testing proved satisfactory.

Using the information displayed in the ***TranscriptImage***, it was possible to trace the design synthesis process, and therefore establish if all necessary steps as indicated in the section 3.8 were carried out. The most common error that occurred during the development was that the application suddenly stopped during the synthesis process without any apparent reason. Because all values are still stored in the slots of all objects and the display of rule and function names in the ***TranscriptImage*** it was possible to determine the last action and which should have been the next one. The cause for this error usually is that no rule was applicable in the given case; this is either due to not including an appropriate condition in the IF-part of a rule that should have been fired, or not setting or re-setting values in specific slot.

Another problem in testing the design synthesis feature is the fact that there is not only one thermowell design for a given application, but a number of possible designs. It is therefore not possible to use the examples in Murdock (1959) and Gibson (1995), specify the given process conditions and expect that the procedure implemented in the expert system will design a thermowell with the same characteristics as the ones that were analysed in the papers. However, the design that is proposed by the expert system must pass the thermowell analysis. It is therefore possible to analyse every design that has been established by the expert system and verify the correctness of the implemented design procedure and the necessary rules and functions. Additionally, due to the way the design synthesis process was implemented in the expert system, it can be expected that the same design will be produced for two identical applications. If changes are made to the application, for example to improve the time it takes to achieve a design, the design process can be verified again by using the examples for which design have already been produced.

The final application has been thoroughly tested. However, as can be said for any form of software development, it is not possible to test software 100%. There is always a chance that a combination of values or other circumstances will cause problems. It is therefore important that any error or problem is reported, together with the input made previous to the problem's occurrence. If the error is reproducible then it is possible to solve the problem. A problem that cannot be reproduced is almost impossible to remove, however.

4. Conclusions

The main aim of developing an expert system was to provide British Rototherm with a facility for routine design and analysis of thermowells. In order to do so, appropriate methods and procedures for the design of thermowells were established. The equations used to calculate the natural frequency of the different thermowells have been validated in practical tests and they predict the frequency closer to the measured frequency than other equations found in text books. Subsequently, these equations were implemented in the expert system, together with methods dealing with the pressure, stress, process condition and thermal considerations.

The 'Thermowell Design Manager' in its present state is capable of analysing thermowell designs that have been provided by the client and establish a suitable design for a given application. Additionally, the system can also determine the influence of the thermowell on the temperature measurement and estimate the thermowell's cost.

Unlike for the methods found during the literature review, there are no restrictions to the dimensions of the tapered thermowells that can be analysed by the expert system. Furthermore, the expert system can analyse and design not only tapered thermowells, but also parallel and reduced parallel thermowells. The combination of these features gives British Rototherm an advantage over its competitors. Queries from clients can now be dealt with as soon as details about the application have been provided, without having to contact subcontractors that formerly carried out the necessary analysis. This enables Rototherm to respond quickly to customer enquires and orders, and at the same time saves additional costs caused by involving subcontractors.

5. Recommendations for Future Work

The expert system in its current state has a sound technical base for the design synthesis and analysis. However, the structure of the application could be simplified. For example, the class **ProcessConditions** is not necessary because the information is stored in class **Order** as well. The instances ***RTD***, ***Thermocouple*** and ***Others*** are also not required as they are only required to allow the user to make a selection; it is therefore possible to store the names of these sensors in an appropriate slot, for instance in class **Sensor**.

One area of the 'Thermowell Design Manager' has to be developed further: the material selection process during the thermowell synthesis. This requires a link to external materials and fluids databases which contain all the relevant information necessary to make a decision on a suitable material for a given fluid at specified temperatures and pressures. At the same time, thermal properties of the fluid and material have to be given as well. According to UK Steel (1998), such comprehensive databases currently do not exist.

The thermal considerations during the thermowell synthesis could be included in the design synthesis process by incorporating rules that can alter the design in order to minimise the measurement error. Similarly, the feature that establishes the thermowell price could also be extended in order to provide a more cost-effective design.

Additionally, the analysis feature could be modified in such a way that, if the analysis fails, a thermowell suitable for the application will be designed.

Apart from the technical side the user friendliness of the 'Thermowell Design Manager' should be improved. The exact improvements necessary will only surface once the application has been in use for a few months. At the same time, this might show some errors within the software that have not been encountered during the test phase. These errors can then be eliminated.

Also implemented should be the 'Help' facility which would allow a novice user to work with the expert system. Additionally, a customer database could be linked with the 'Thermowell Design Manager' to automatically consider discounts for regular customers.

An advanced feature that could prove useful in connection with the expert system is a link to the CAD software used at British Rototherm. This would make it possible to analyse a thermowell from a given drawing, and the specifications for the thermowell are not restricted to the given input fields in the expert system. It would therefore be possible to determine the vibration, pressure and stress criteria for thermowells of unusual shape; for instance, a thermowell which is parallel for most of the length but tapered at the last, say, 25% of the immersion length could be analysed. Such uncommon designs are currently not considered by the expert system.

Finally, the possibility to develop several conceptual designs for a given application and from that choosing the most suitable design needs to be investigated.

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7. Copyright Notes

APPENDIX I: Calculation of L_{max} according to Murdock/PTC and Roughton

$A := 1\cdot in$ $\qquad B := \frac{13}{16}\cdot in$ $\qquad d := \frac{7}{16}\cdot in$ $\qquad t := \overrightarrow{\frac{B-d}{2}}$ $\qquad L := 10\cdot in$

$S := 133.42\,10^{6}\cdot\frac{newton}{m^2}$ $\qquad E := 29.15\,10^{6}\cdot\frac{lb}{in^2}$ $\qquad G := 7968\,\frac{kg}{m^3}$

$v := 20\cdot\frac{m}{sec}$ $\qquad P_O := 1.01325\,10^{5}\cdot\frac{newton}{m^2}$

Air:

$\rho := 1.184\,\frac{kg}{m^3}$ $\qquad \upsilon := \frac{1}{\rho}$

$$y := \left[\frac{4\cdot G}{3\cdot E\cdot\left(\frac{A-B}{L}\right)^4}\cdot\left[\frac{1}{2}\cdot(A-B)^2 \ldots \\ +\left[\frac{1}{4}\cdot\left[3\cdot\frac{B^4}{d^2}-5\cdot d^2-6\cdot\left[B+t\cdot\left(\frac{A-B}{L}\right)\right]^2\right]\cdot\left[\frac{B}{d}\cdot\ln\left[\frac{(B-d)\cdot(A+d)}{(B+d)\cdot(A-d)}\right]+\ln\left(\frac{A^2-d^2}{B^2-d^2}\right)\right]\ldots \\ +\frac{1}{2}\cdot\left[6\cdot\left(B+t\cdot\frac{A-B}{L}\right)^2-7\cdot d^2-\frac{3\cdot B^4}{d^2}\right]\cdot\left[\left[\frac{B}{d}\cdot\operatorname{atan}\left[\frac{d\cdot(B-A)}{d^2+A\cdot B}\right]\right]+\frac{1}{2}\cdot\ln\left(\frac{A^2+d^2}{B^2+d^2}\right)\right]\ldots \\ +2\cdot\left[\frac{B^3}{d^2}-3\cdot\left[B+t\cdot\left(\frac{A-B}{L}\right)\right]\right]\cdot\left[\frac{B}{2}\cdot\ln\left[\frac{\left(B^2+d^2\right)\cdot\left(A^2-d^2\right)}{\left(B^2-d^2\right)\cdot\left(A^2+d^2\right)}\right]+\left[d\cdot\operatorname{atan}\left[\frac{d\cdot(B-A)}{d^2+A\cdot B}\right]\ldots \\ +\frac{d}{2}\cdot\ln\left[\frac{(B-d)\cdot(A+d)}{(B+d)\cdot(A-d)}\right]\right]\right]\right]\right]\right]$$

Use of PTC19.3 tables:

Constant: $\qquad k_f := 2.661\cdot\frac{\sqrt{in^3}}{sec}$

Natural Frequency: $\qquad f_{n.PTC} := \frac{k_f}{L^2}\cdot\sqrt{\frac{E}{G}}$

$f_{n.PTC} = 267.78\cdot Hz$

Use of Deflection-Method:

Deflection: $\qquad y = 3.469\,10^{-6}\cdot m$

$f_{n.Defl} := \overrightarrow{\left(\frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}\right)}$

$f_{n.Defl} = 267.6\cdot Hz$

Calculation of L_{max}:

$tD := \frac{A-d}{2\cdot A}$ $\qquad tD = 0.281$ $\qquad F_A := 1$ $\qquad k_x := \frac{1.698(A+2\cdot B)\cdot A}{A^4-d^4}$ $\qquad k_2 := \sqrt{\frac{2}{F_A\cdot k_x}}$

$k_A := \frac{A^2}{A^2-d^2}$ $\qquad k_3 := \left[F_A\cdot\left(k_A-1\right)\right]$

$f_w := 0.22\cdot\frac{v}{B}$ $\qquad f_w = 213.204\cdot Hz$ $\qquad r := \frac{f_w}{f_{n.Defl}}$ $\qquad r = 0.797$ $\qquad F_M := \overrightarrow{\frac{r^2}{1-r^2}}$

$L_{max} := \left[\frac{k_2}{v}\cdot\sqrt{\frac{\upsilon\cdot\left(S-k_3\cdot P_O\right)}{1+F_M}}\right]$ $\qquad L_{Roughton} := \left[\sqrt{\frac{k_f}{1.25\cdot f_w}}\cdot\left(\frac{E}{G}\right)^{\frac{1}{4}}\right]$

$L_{max} = 5.356\,m$ $\qquad L_{Roughton} = 0.255\,m$ $\qquad L = 0.254\,m$

Propanol:

$\rho := 804.66\frac{kg}{m^3}$ $\quad \upsilon := \frac{1}{\rho}$

Calculation of L_{max}:

$tD := \frac{A - d}{2 \cdot A}$ $\quad tD = 0.281$ $\quad F_A := 1$ $\quad k_x := \frac{1.698(A + 2 \cdot B) \cdot A}{A^4 - d^4}$ $\quad k_2 := \sqrt{\frac{2}{F_A \cdot k_x}}$

$k_A := \frac{A^2}{A^2 - d^2}$ $\quad k_3 := \left[F_A \cdot (k_A - 1)\right]$

$f_w := 0.22 \cdot \frac{v}{B}$ $\quad f_w = 213.204 \cdot Hz$ $\quad r := \frac{f_w}{f_{n.Defl}}$ $\quad r = 0.797$ $\quad F_M := \overrightarrow{\frac{r^2}{1 - r^2}}$

$L_{max} := \left[\frac{k_2}{v} \cdot \sqrt{\frac{\upsilon \cdot (S - k_3 \cdot P_O)}{1 + F_M}}\right]$ $\quad L_{Roughton} := \left[\sqrt{\frac{k_f}{1.25 \cdot f_w}} \cdot \left(\frac{E}{G}\right)^{\frac{1}{4}}\right]$

$L_{max} = 0.205\,m$ $\quad L_{Roughton} = 0.255\,m$ $\quad L = 0.254\,m$

APPENDIX II: Methods for the calculation of the natural frequency of cantilever beams

This section discusses briefly the other approaches used to determine the natural frequency of cantilever beams.

Vibration of Continuous Systems

The continuous systems under consideration are assumed to be homogenous and isotropic, obeying Hooke's law of elasticity within elastic limits. The systems have an infinite umber of degrees of freedom as an infinite number of coordinates is required in order to describe the position of every point of the body.

During free vibration, every particle of the body performs simple harmonic motion at its natural frequency, with each particle passing through its equilibrium position at the same time.

Euler equations are available for beams experiencing lateral vibration caused by the beam's own weight. Looking at one element of the beam the following loads can be identified (Figure AII-1):

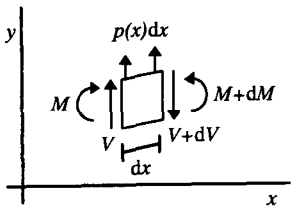


Figure AII-1: Beam element

with V: shear moment

M: bending moment

$p(x)$: load per unit length (in this case caused by the weight)

Forces in y-direction:

$$\mathrm{d}V - p(x)\mathrm{d}x = 0$$

Moments:

$$\mathrm{d}M - V\mathrm{d}x - \tfrac{1}{2}p(x)(\mathrm{d}x)^2 = 0$$

This results in the following relationships:

$$\frac{\mathrm{d}V}{\mathrm{d}x} = p(x); \quad \frac{\mathrm{d}M}{\mathrm{d}x} = V; \quad \frac{\mathrm{d}^2 M}{\mathrm{d}x^2} = \frac{\mathrm{d}V}{\mathrm{d}x} = p(x)$$

From the basic beam theory it can be seen that the bending moment is related to the curvature by the flexure equation:

$$M = EJ\frac{\mathrm{d}^2 y}{\mathrm{d}x}$$

$$\therefore \frac{\mathrm{d}^2}{\mathrm{d}x^2}(EJ\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}) = p(x)$$

Looking at a beam vibrating about its equilibrium position under its own weight, the load per unit length is the same as the inertia load due to its mass and acceleration. If harmonic motion is assumed it follows:

$$p(x) = \rho\omega^2 y$$

with ρ being the mass per unit length.

The general solution for the established equation

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} - \beta^4 y = 0$$

is

$$y = A\cosh\beta x + B\sinh x + C\cosh x + D\sin\beta x$$

which results in

$$f_n = \beta_n^2\sqrt{\frac{EJ}{\rho}} = (\beta_n L)^2\sqrt{\frac{EJ}{\rho L^4}}$$

The value for β_n depends on the boundary conditions for each beam and is readily available in the literature. For the fundamental frequency of cantilever beams $(\beta_n L)^4$=3.52.

Formulas for natural frequency and mode shape

R D Blevins (1979) derived equations for the calculation of both the natural frequency and the mode shape of beams. An equation is proposed that determines the natural frequency of a beam, depending on its boundary conditions:

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\sqrt{EJ/m}$$

with i: mode

λ_i: parameter depending on boundary conditions

m: $^{\text{mass}}/_{\text{unit length}}$

For a cantilever beam, the values of λ_i are:

Table AII-1: Values of λ_i for cantilever beams

Mode i	λ_i
1	1.87510407
2	4.69409113
3	7.85475744
4	10.99554073
5	14.13716839
>5	$(2i-1)^{\pi}/_2$

As was discussed previously, thermowells can be modelled as cantilever beams; therefore this approach can be used for parallel thermowells using λ_1 = 1.87510407 to establsish f_n.

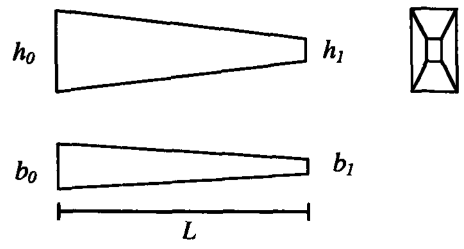
For tapered thermowells, the equation proposed for truncated linearly tapered has to be used. This equation and the values for λ_i were developed for a rectangular cross section, but Blevins suggests that they should provide a good approximation for other closed cross-section such as circular cross-sections.

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{EJ_0}{\mu A_0}}$$

with J_0: second moment of area at widest point along span

A_0: area of cross-section at widest point along span

μ: density of material



h: height of beam
b: width of beam
L: length of beam

Figure AII-2: truncated linearly tapered beam

Again, λ_i depends on the boundary conditions and also on the taper of the beam. To establish λ_i figures for different boundary conditions and tapers are provided in Blevins (1979). The ratios h_0/h_1 and b_0/b_1 specify the taper of the cross-section. For circular cross-sections, and therefore for thermowells, the height and width are equal and therefore the ratios are equal, too. The value of λ_1 for the tapered thermowells used in the tests is λ_1 = 1.941; it was taken from Figure 8-18 on page 163 in Blevins 1979 using the ratio $^A/_B$=1.222.

Integral equation approach

Penny and Reed (1971) use integral equations and the Hilbert-Schmidt method to establish equations that allow the calculation of the natural frequency of various beams. The Hilbert-Schmidt method allows finding an approximation to the lowest natural frequency of a vibrating beam together with an error bound.

Applying this approach to a uniform cantilever, a lower and upper bound for the natural circular frequency were established:

$$3.52\sqrt{\frac{EJ}{\rho AL^4}} < f_1 < 3.65\sqrt{\frac{EJ}{\rho AL^4}}$$

A general equation was derived by the authors, which uses a constant C to represent different cases:

$$f_1 = C\sqrt{\frac{EJ}{\rho AL^4}}$$

The value of C can be established from a table for different cases given in the paper. The table's case 4 is closest to a tapered thermowell, resulting in C=8.65.

Rayleigh's method

This method is the generally accepted approach to find an approximation for the fundamental frequency of a system (Harris 1994). Lord Rayleigh's argument is that the natural frequency can be calculated from the energies associated with a body's vibration. When an elastic system without damping vibrates in its fundamental normal mode, each part of the system executes simple harmonic motion about its equilibrium position. Therefore, the lateral vibration of a beam can be expressed with the relationship $y = X(x)\sin\omega t$, with $X(x)$ being a function of the distance x along the beam.

If the deflection of the body from the equilibrium position is at its maximum then all parts of the body are motionless, therefore all the energy of the body associated with vibration is in the form of elastic strain energy. When the body passes through its equilibrium position then all of the vibration energy is in the form of kinetic energy. For conversation of energy the kinetic energy must equal the strain energy, i.e. $E_{\text{kinetic}} = E_{\text{strain}}$. As was stated earlier, Rayleigh argues that when the two energies involved are computed and equated then the resulting equation can be solved for f_n.

For a cantilever beam under uniform load, the maximum strain energy can be calculated with

$$V = {}^{EJ}\!/_{2}\int_0^L \left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^2 \mathrm{d}x = {}^{8}\!/_{5}\frac{EJq^2}{L^3}$$

with q: load per unit length

The maximum kinetic energy is

$$T = \frac{\omega_n^2 \gamma A}{2g}\int_0^L y^2 \mathrm{d}x = {}^{52}\!/_{405}\frac{\omega_n^2 \gamma A L q^2}{g}$$

with γ: density

A: cross-sectional area

Equating those two energies results in a relationship for the circular frequency

$$\omega_n = {}^{3.53}\!/_{L^2}\sqrt{\frac{EJg}{\gamma A}}$$

APPENDIX III: Influence of wall thickness at the tip on the natural frequency

Thermowell properties:

$$E := 201 \cdot 10^9 \cdot \frac{newton}{m^2} \qquad G := 7970 \frac{kg}{m^3}$$

$$A := 16 \cdot mm \quad B := 16 \cdot mm \qquad s := 135 \cdot mm \qquad d := 8 \cdot mm \qquad L := 135 \cdot mm$$

$$t := \begin{bmatrix} 3 \\ 10 \\ 50 \\ 80 \\ 100 \\ 125 \end{bmatrix} \cdot mm$$

Moment-area method:

$$x := 0 \cdot m, 0.0002m .. L$$

$$J1 := \overrightarrow{\left[\frac{\pi}{64} \cdot \left(B^4 - d^4\right)\right]} \qquad J2 := \overrightarrow{\left[\frac{\pi}{64} \cdot \left(A^4 - d^4\right)\right]} \qquad J3 := \overrightarrow{\left(\frac{\pi}{64} \cdot B^4\right)}$$

$$q1 := \overrightarrow{\frac{\pi \cdot G \cdot g \cdot \left(B^2 - d^2\right)}{4}} \quad q2 := \overrightarrow{\frac{\pi \cdot G \cdot g \cdot \left(A^2 - d^2\right)}{4}} \quad q3 := \overrightarrow{\frac{\pi \cdot G \cdot g \cdot B^2}{4}}$$

$$f(x) := (x - s) \quad b(x) := if(f(x) \le 0, 0, f(x))$$

$$k(x) := if(f(x) \le 0, 0, 1)$$

$$p(x) := \left(x - t_0\right) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3 \cdot (1 - n(x)) + J1 \cdot (n(x) - k(x)) + k(x) \cdot J2)$$

$$EJy'''(x) := \overrightarrow{(q3 \cdot (1 - n(x)) + (q1 \cdot (n(x) - k(x)) + q2 \cdot k(x)))}$$

$$EJy''(x) := \overrightarrow{(q3 \cdot (x - a(x)) + (q1 \cdot (a(x) - b(x)) + q2 \cdot b(x)))}$$

$$EJy'(x) := \overrightarrow{\left[\frac{1}{2} \cdot \left[q3 \cdot \left(x^2 - a(x)^2\right) + q1 \cdot \left(a(x)^2 - b(x)^2\right) + q2 \cdot b(x)^2\right]\right]}$$

$$Ey'(x) := \overrightarrow{\frac{EJy'(x)}{I(x)}}$$

$$M_y := \overrightarrow{\int_0^L x \cdot Ey'(x)\, dx} \qquad M_y = 1.654 \cdot 10^5 \cdot kg \cdot sec^{-2} \qquad y := \frac{M_y}{E} \quad y = 0.000822911 \cdot mm$$

$$f_{n.MA_0} := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{g}{y}}$$

$$p(x) := \left(x - t_1\right) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3 \cdot (1 - n(x)) + J1 \cdot (n(x) - k(x)) + k(x) \cdot J2)$$

$$EJy'''(x) := (q3 \cdot (1 - n(x)) + (q1 \cdot (n(x) - k(x)) + q2 \cdot k(x)))$$

$$EJy''(x) := (q3 \cdot (x - a(x)) + (q1 \cdot (a(x) - b(x)) + q2 \cdot b(x)))$$

$$EJy'(x) := \left[\frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]\right]$$

$$Ey'(x) := \frac{EJy'(x)}{I(x)}$$

$$M_y := \int_0^L x\cdot Ey'(x)\,dx \qquad M_y = 1.723\cdot 10^5 \cdot kg\cdot sec^{-2} \qquad y := \frac{M_y}{E} \quad y = 0.000857431\cdot mm$$

$$f_{n.MA_1} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}$$

$$p(x) := \left(x - t_2\right) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3\cdot(1 - n(x)) + J1\cdot(n(x) - k(x)) + k(x)\cdot J2)$$

$$EJy'''(x) := (q3\cdot(1 - n(x)) + (q1\cdot(n(x) - k(x)) + q2\cdot k(x)))$$

$$EJy''(x) := (q3\cdot(x - a(x)) + (q1\cdot(a(x) - b(x)) + q2\cdot b(x)))$$

$$EJy'(x) := \left[\frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]\right]$$

$$Ey'(x) := \frac{EJy'(x)}{I(x)}$$

$$M_y := \int_0^L x\cdot Ey'(x)\,dx \qquad M_y = 2.009\cdot 10^5 \cdot kg\cdot sec^{-2} \qquad y := \frac{M_y}{E} \quad y = 0.000999659\,mm$$

$$f_{n.MA_2} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}$$

$$p(x) := \left(x - t_3\right) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3\cdot(1 - n(x)) + J1\cdot(n(x) - k(x)) + k(x)\cdot J2)$$

$$EJy'''(x) := (q3\cdot(1 - n(x)) + (q1\cdot(n(x) - k(x)) + q2\cdot k(x)))$$

$$EJy''(x) := (q3\cdot(x - a(x)) + (q1\cdot(a(x) - b(x)) + q2\cdot b(x)))$$

$$EJy'(x) := \left[\frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]\right]$$

$$Ey'(x) := \frac{EJy'(x)}{I(x)}$$

$$M_y := \int_0^L x\cdot Ey'(x)\,dx \qquad M_y = 2.103\cdot 10^5 \cdot kg\cdot sec^{-2} \qquad y := \frac{M_y}{E} \quad y = 0.001046225\,mm$$

$$f_{n.MA_3} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}$$

$$p(x) := \left(x - t_4\right) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3\cdot(1 - n(x)) + J1\cdot(n(x) - k(x)) + k(x)\cdot J2)$$

$$EJy'''(x) := (q3\cdot(1 - n(x)) + (q1\cdot(n(x) - k(x)) + q2\cdot k(x)))$$

$$EJy''(x) := (q3\cdot(x - a(x)) + (q1\cdot(a(x) - b(x)) + q2\cdot b(x)))$$

$$EJy'(x) := \left[\frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]\right]$$

$$Ey'(x) := \frac{EJy'(x)}{I(x)}$$

$$M_y := \int_0^L x \cdot Ey'(x)\,dx \qquad M_y = 2.112 \cdot 10^5 \cdot kg \cdot sec^{-2} \qquad y := \frac{M_y}{E} \qquad y = 0.001050696mm$$

$$f_{n.MA_4} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}$$

$$p(x) := (x - t_5) \qquad n(x) := if(p(x) \le 0, 0, 1)$$

$$a(x) := if(p(x) \le 0, 0, p(x))$$

$$I(x) := (J3\cdot(1 - n(x)) + J1\cdot(n(x) - k(x)) + k(x)\cdot J2)$$

$$EJy'''(x) := (q3\cdot(1 - n(x)) + (q1\cdot(n(x) - k(x)) + q2\cdot k(x)))$$

$$EJy''(x) := (q3\cdot(x - a(x)) + (q1\cdot(a(x) - b(x)) + q2\cdot b(x)))$$

$$EJy'(x) := \left[\frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]\right]$$

$$Ey'(x) := \frac{EJy'(x)}{I(x)}$$

$$M_y := \int_0^L x \cdot Ey'(x)\,dx \qquad M_y = 2.064 \cdot 10^5 \cdot kg \cdot sec^{-2} \qquad y := \frac{M_y}{E} \qquad y = 0.001026687mm$$

$$f_{n.MA_5} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}$$

Standard approach:

$$J := \left[\frac{\pi}{64}\cdot\left(B^4 - d^4\right)\right] \qquad q1 := \frac{\pi\cdot G\cdot g\cdot\left(B^2 - d^2\right)}{4} \qquad q2 := \overrightarrow{\frac{\pi\cdot G\cdot g\cdot B^2}{4}}$$

$$y := \overrightarrow{\left[\frac{1}{E\cdot J}\cdot\left[\frac{q1}{24}\cdot\left[L^4 - (L - (L - t))^4\right] + \frac{q2}{24}\cdot(L - (L - t))^4 + \frac{1}{6}\cdot(q1\cdot(t - L) - q2\cdot t)\cdot L^3 \ldots \atop + \frac{1}{2}\cdot\left[\frac{q1}{2}\cdot L^2 + \frac{t^2}{2}\cdot(q1 - q2) + L\cdot t\cdot(q2 - q1)\right]\cdot L^2\right]\right]}$$

$$f_n := \overrightarrow{\frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y}}}$$

Results:

$$f_{n.MA} = \begin{bmatrix} 549.419 \\ 538.246 \\ 498.488 \\ 487.268 \\ 486.23 \\ 491.882 \end{bmatrix}\cdot Hz \qquad f_n = \begin{bmatrix} 549.419 \\ 538.246 \\ 498.19 \\ 485.348 \\ 481.72 \\ 480.442 \end{bmatrix}\cdot Hz \qquad t = \begin{bmatrix} 3 \\ 10 \\ 50 \\ 80 \\ 100 \\ 125 \end{bmatrix}\cdot mm \qquad \frac{t}{L} = \begin{bmatrix} 2.222 \\ 7.407 \\ 37.037 \\ 59.259 \\ 74.074 \\ 92.593 \end{bmatrix}\cdot\%$$

Appendix IV: Stepped Parallel Thermowells

$E := 201\cdot 10^{9}\cdot\frac{newton}{m^2}$ $G := 7970\cdot\frac{kg}{m^3}$

$A := 16\cdot mm$ $B := 12\cdot mm$ $d := 8\cdot mm$ $t := 3\cdot mm$ $L := 205\cdot mm$

$r := 1, 2 .. 205$

$s_0 := 0\cdot mm$

$s_r := s_{r-1} + 1\cdot mm$

$x := 0\cdot mm, 0.1\cdot mm .. L$

$w := 0, 1 .. 204$

$J1 := \frac{\pi}{64}\cdot\left(B^4 - d^4\right)$ $J2 := \frac{\pi}{64}\cdot\left(A^4 - d^4\right)$ $J3 := \frac{\pi}{64}\cdot B^4$

$q1 := \frac{\pi}{4}\cdot G\cdot g\cdot\left(B^2 - d^2\right)$ $q2 := \frac{\pi}{4}\cdot G\cdot g\cdot\left(A^2 - d^2\right)$ $q3 := \frac{\pi}{4}\cdot G\cdot g\cdot B^2$

$f(x) := x - s_w$ $p(x) := x - t$

$b(x) := if(f(x) \le 0, 0, f(x))$ $n(x) := if(p(x) \le 0, 0, 1)$

$k(x) := if(f(x) \le 0, 0, 1)$ $a(x) := if(p(x) \le 0, 0, p(x))$

$I(x) := J3\cdot(1 - n(x)) + J1\cdot(n(x) - k(x)) + J2\cdot k(x)$

$EJy'''(x) := q3\cdot(1 - n(x)) + (q1\cdot(n(x) - k(x)) + q2\cdot k(x))$

$EJy''(x) := q3\cdot(x - a(x)) + (q1\cdot(a(x) - b(x)) + q2\cdot b(x))$

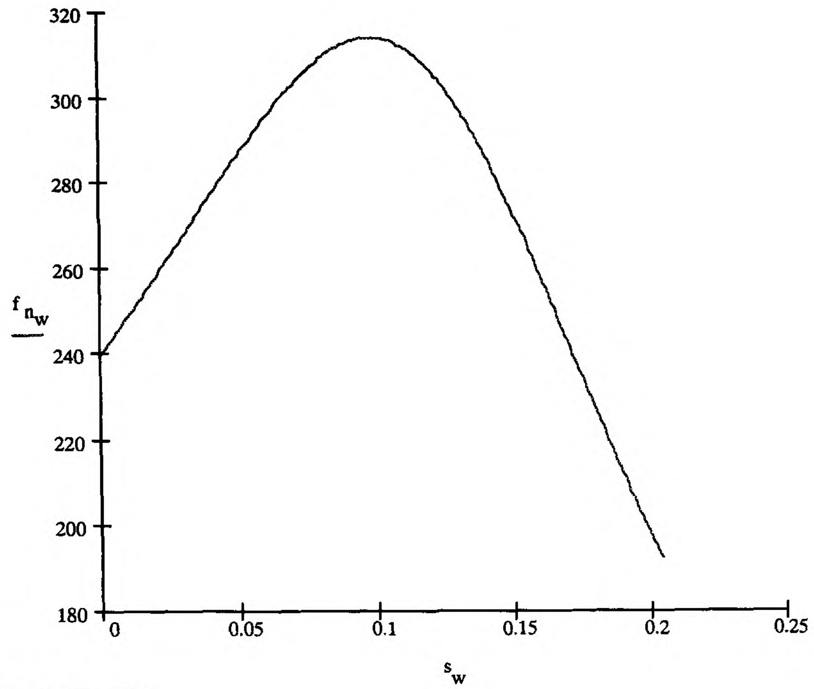
$EJy'(x) := \frac{1}{2}\cdot\left[q3\cdot\left(x^2 - a(x)^2\right) + q1\cdot\left(a(x)^2 - b(x)^2\right) + q2\cdot b(x)^2\right]$

$Ey'(x) := \frac{EJy'(x)}{I(x)}$

$M_{y_w} := \int_0^L x\cdot Ey'(x)\,dx$

$y := \overrightarrow{\frac{M_y}{E}}$

$f_{n_w} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{g}{y_w}}$



$f_n =$

	0
87	311.713
88	312.052
89	312.328
90	312.726
91	312.901
92	313.046
93	313.254
94	313.378
95	313.495
96	313.647
97	313.645
98	313.69
99	313.599
100	313.639
101	313.651

APPENDIX V: Introduction to KAPPA-PC

KAPPA-PC is an *object oriented* development system. This is adapted from the view that the world is a collection of objects which have certain characteristics, abilities, parts and relationships between each other. Thermowells, for example, are objects. They have attributes such as bore diameter and immersion length; their components are the process connection, the sensor connection and the stem. It is therefore necessary to create objects when building an application using KAPPA-PC. For this project, objects such as the thermowell itself and the customer will be used.

Objects in KAPPA-PC are divided into two categories: *classes* and *instances*. A class is a general object and can represent a group or collection; instances are specific objects and would, for example, represent the member of a group. Applying this to the thermowell application again, thermowells can be considered as a class and a parallel thermowell, for example, is an instance of the class thermowells. An instance of a given class will inherit all the attributes of the class. These are the characteristics which are common to all instances of a class. For example, every thermowell has an immersion length and a bore diameter. The difference between the three types of thermowells are the diameters necessary to describe them, or an additional reduced length. These attributes would only be present in the appropriate instance. This object oriented structure is referred to as a *hierarchy*. In KAPPA-PC this hierarchy is represented in graphical form; Figure AV-1 shows a small hierarchy using the thermowell example as it would be displayed in KAPPA-PC's *object browser* (the utility used for this purpose). Note that the class **Root** is present in all KAPPA-PC applications.

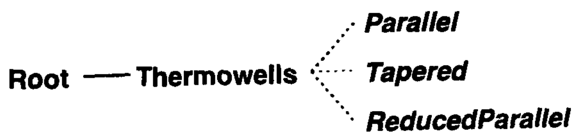


Figure AV-1: Hierarchy for thermowells

The attributes used to describe the objects are stored in *slots* in each object. For example, the class **Thermowells** could have a slot called 'immersion length' which specifies the thermowell's immersion length. Similarly, the instance ***ReducedParallel*** could have a slot called 'reduced length' to describe that length.

The object orientation is also used internally by KAPPA-PC. The windows and images such as buttons, input fields etc. that are used to create the graphical user interface are also represented by instances of the appropriate class. For example, the windows (referred to as *session windows* in the KAPPA-PC environment) are instances of the class **SESSION**.

KAPPA-PC has its own programming language KAL (KAPPA-PC Application Language), which offers standard routines such as If...Then... or While loops and KAPPA-PC specific non-standard routines such as SetSlotValue(Object, Slot, Value), which defines the value of a slot in the specified object. The structure of KAL is similar to that of C or C++, because an application consists of a set of function calls. These functions are either existing KAL functions or user-defined functions. Figure AV-2 shows a short example for a KAL routine. This routine has to be in a user-defined function, for example with the name Example().

```
ShowWindow(Session1);
SetValue(Thermowells:Length, 150);
DisplayText(TranscriptImage1, Thermowells:Length);
```

Figure AV-2: Example for a KAL routine

This routine displays a window called ***Session1*** which contains a field that can display text. The value of slot Length in object **Thermowells** is set to 150, which is then displayed in the field for text output, ***TranscriptImage1***. The function can be called from other functions, or it can be linked to a button image which will call the function when the appropriate button is clicked by the user.

It is also possible to use rules, which are If...Then... statements in KAPPA-PC. Both parts of a rule can consist of a series of functions that have to be evaluated to determine whether the If... part is true and what actions have to be carried out in that case (defined in the Then... part of the rule). As an example for a rule the vibration criterion is used. It states that the frequency ratio between wake frequency and natural frequency of a thermowell has to be less than (or equal to) 0.8 for a thermowell to pass the analysis. In fact, this is already a rule. A shorter version of this rule is: 'If the thermowell's frequency ratio <= 0.8 then the thermowell has passed the analysis'. Figure AV-3 shows what this rule (with the name **Passed?**) would look like in KAPPA-PC.

Passed?
If Thermowells:FrequencyRatio <= 0.8
Then Thermowells:Passed = Yes;

Figure AV-3: Example for a rule

This rule checks whether the frequency ratio (which has been calculated before this rule was fired and the result has been stored in slot 'FrequencyRatio') is smaller than or equal to 0.8. If this is the case, then the value of slot 'Passed' is set to Yes. Alternatively, a message could be displayed that states that the vibration criterion has been satisfied.

Using so-called *patterns*, rules can be applied to all instances of a class without having to identify the instances individually. For example, if all instances of class **Thermowells** have to be checked whether they passed the vibration analysis, then the following rule can be used:

Passed? [wells | Thermowells]
If wells:FrequencyRatio <= 0.8
Then wells:Passed = Yes;

Figure AV-4: Example for a rule using patterns

In this case, the variable **wells** is replaced with each instance name of the class **Thermowells**.

KAPPA-PC supports both forward and backward chaining. There are four different modes available for forward chaining (depth-first, breadth-first, selective, best-first) and a goal can be used for both forward and backward chaining. The functions to start chaining allow the programmer to specify individual rules that have to be used, complete rule sets (a list of rules stored in a slot) or all rules defined in the application. If goals are used then chaining will stop as soon as the conditions in the goal have been satisfied. It is mandatory to use a goal when backward chaining.

Appendix VI: The Monju Leak (Nedderman 1997)

On 8th December 1995, a sodium leak occurred at the prototype Monju fast reactor, caused by a broken thermocouple on a secondary coolant piping. The tip of the reduced-parallel thermowell used in the application had broken off at the point of the step change in diameter. This allowed sodium to leak through the gap between the thermocouple and the thermowell, resulting in the reactor being shut down.

Numerous theoretical and experimental investigations carried out by PNC (the Power Reactor and Nuclear Fuel Development Corp) and its contractors showed that the incident was caused by flow-induced vibration of the thermowell.

The design of the thermowells for the reactor were approached as an instrumentation problem, because a fast response time was essential for some of the thermowells in the cooling loops. Therefore, the designers selected a long, thin walled thermowell, without taking any mechanical aspects into consideration.

Following the investigations the use of tapered thermowells is being considered, together with an improved sodium detector. Fitting these new thermowells, together with getting the necessary approvals, was expected to take up to one year, followed by an extensive period of testing during which numerous flow-induced vibration checks would be made. This rework also means a considerable financial effort for all the parties involved in the project.

However, the incident at the Monju reactor has again given rise to discussions about the safety of nuclear energy, which is a serious political issue. As a result of the accident, approval for the first APWR power plants at the Tsuruga site have indefinitely been delayed.